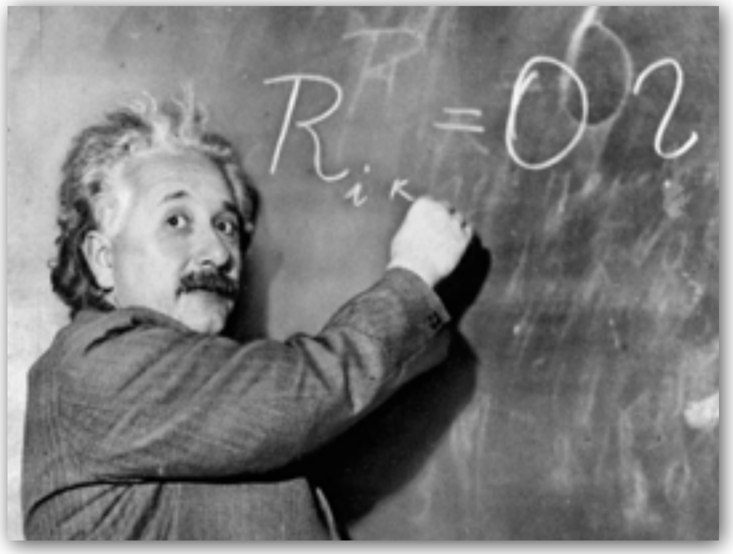  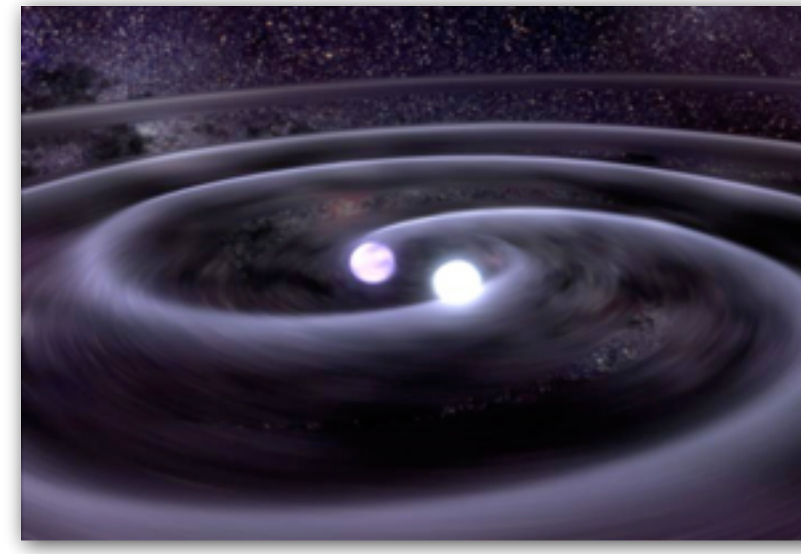

Understanding the relativistic 2-body problem: Gravitational Waves and Numerical Relativity

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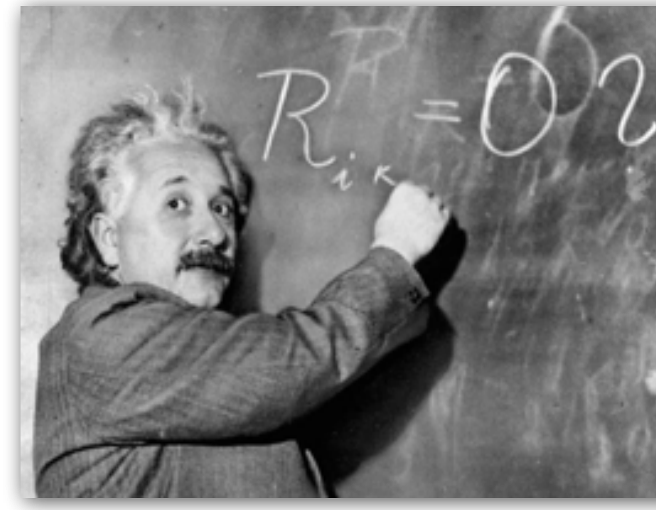
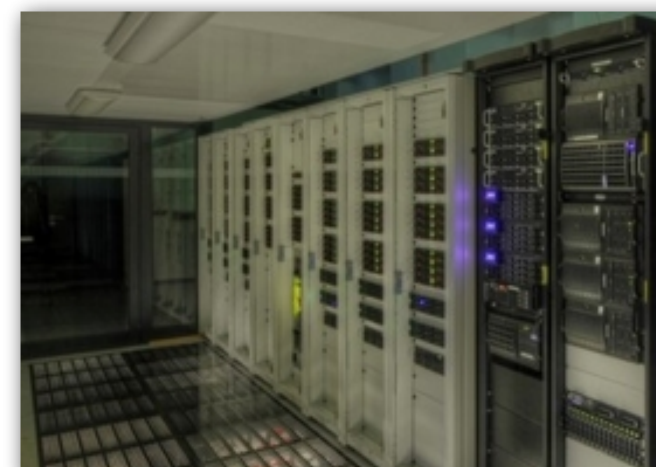
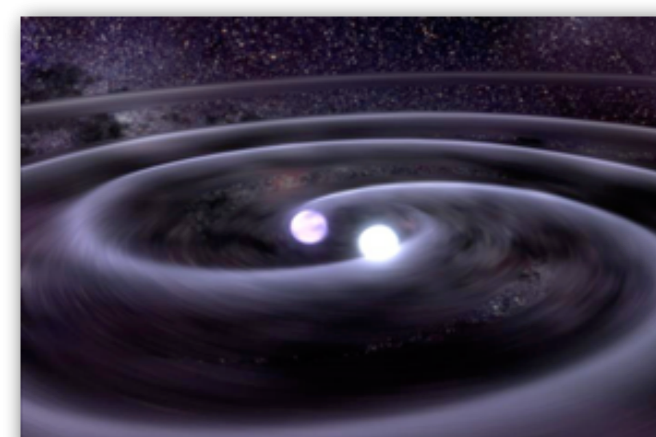
Modern Aspects of Gravitational Theory
Jena, 31st May, 2017

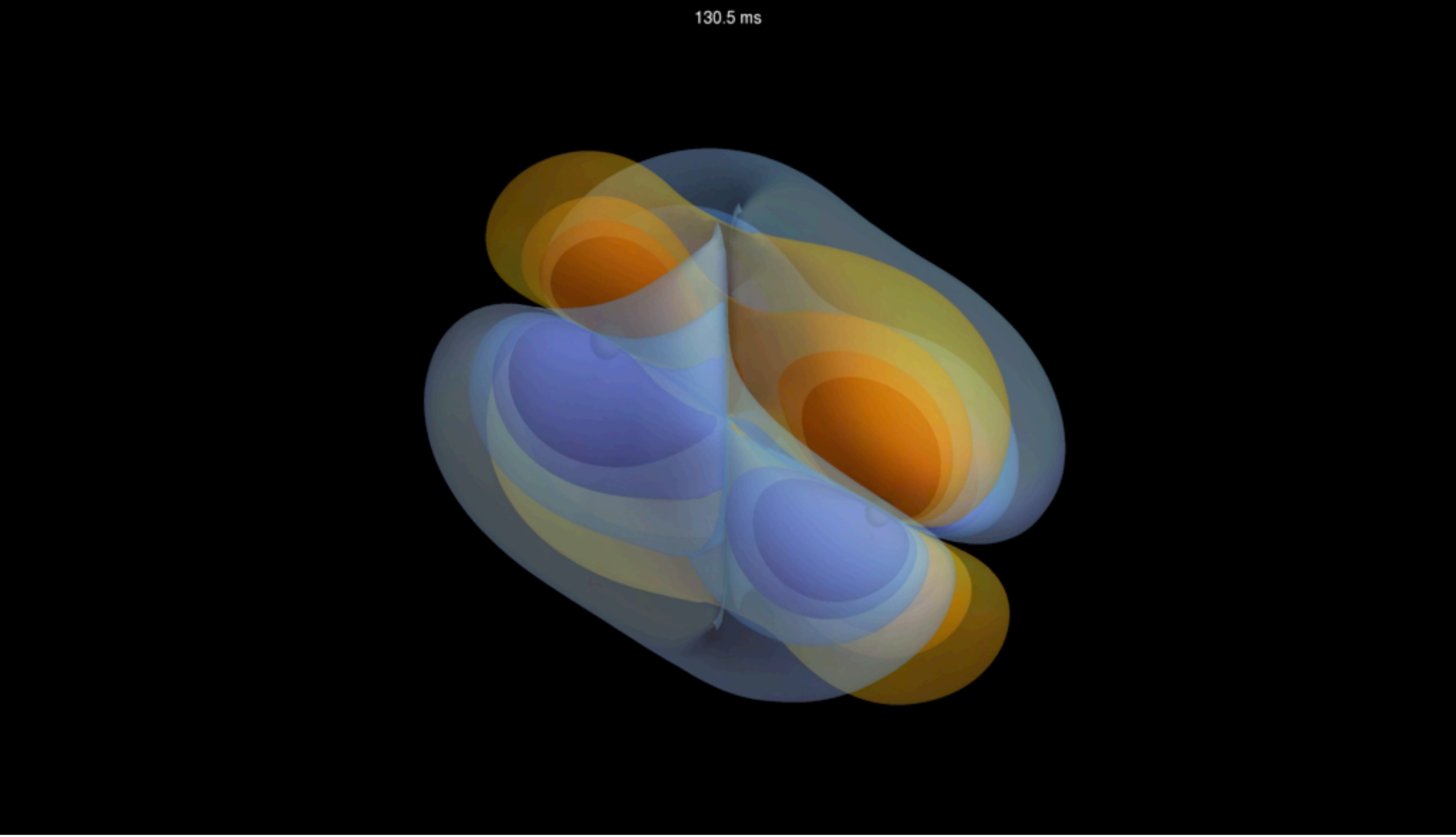
Overview

- Numerical Relativity
    - Physics, mathematics, numerics, computing, software
- Gravitational waves
    - Waveform modelling and LIGO
    - Waveforms from eccentric binaries
- Outlook







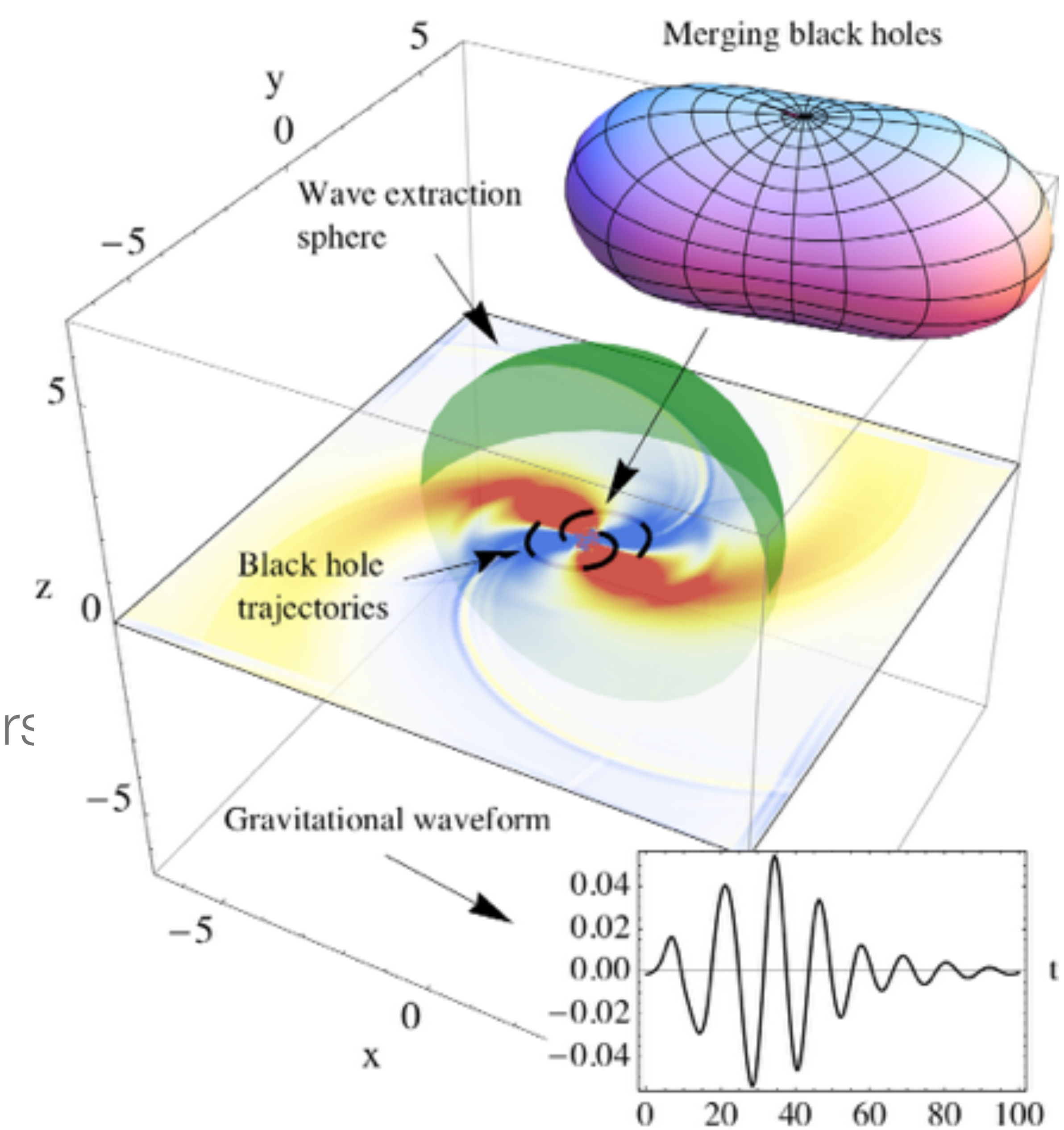


1. Numerical Relativity

Open source simulation of merger of GW150914
[Wardell, IH, Bentivegna]
einsteintoolkit.org/gallery/bbh

Numerical Relativity: Physics

- How does matter and geometry **evolve in time** in General Relativity?
- Some highlights:
  - Binary black hole and neutron star mergers: **test GR** and **high density** physics
  - Supernova core collapse
  - Gravitational wave templates for detectors
  - Cosmology: e.g. how does **light propagate** in an inhomogeneous spacetime? *[Bentivegna, Korzynzki and IH, 2016]*
  - Mathematical relativity (singularity theorems), etc



Compact binary simulation in NR

Numerical Relativity: Maths

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

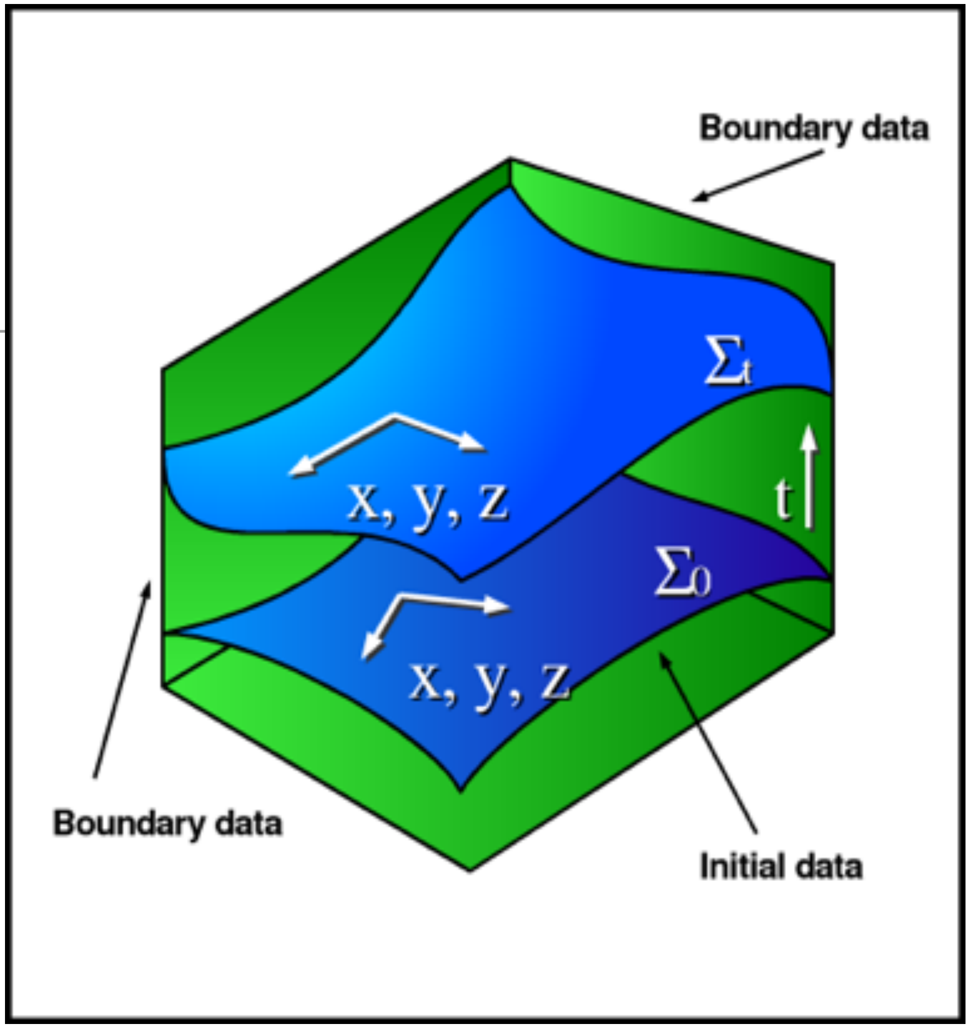
- 10 coupled nonlinear 2nd order partial differential equations:

$$\begin{aligned} {}^{(4)}R_{\mu\nu} &\equiv \frac{1}{2} g^{\sigma\rho}(g_{\sigma\nu,\mu\rho} + g_{\mu\rho,\sigma\nu} - g_{\sigma\rho,\mu\nu} - g_{\mu\nu,\sigma\rho}) \\ &+ g^{\sigma\rho}(\Gamma^{m}{}_{\mu\rho}\Gamma_{m\sigma\nu} - \Gamma^{m}{}_{\mu\nu}\Gamma_{m\sigma\rho}) \\ \Gamma^{\mu}{}_{\nu\sigma} &\equiv \frac{1}{2} g^{\mu\rho}(g_{\rho\nu,\sigma} + g_{\rho\sigma,\nu} - g_{j\sigma,\rho}) \end{aligned}$$

Formulate as **initial boundary value problem** by projecting onto a **foliation** of 3D t=const slices:

$$\frac{\partial}{\partial t} u(t, x^i) = F\left(u(t, x^i), \partial u(t, x^i), \partial^2 u(t, x^i)\right)$$

- ~25 eqs/variables - **complicated**, **nonunique,** issues of **well-posedness**



- Initial data (t=0) evolved forward in time with **evolution equations**
- Also get **constraint equations** on each t=const slice

Einstein equations in 3+1 form

$$\partial_t \hat{\phi}_\kappa = \frac{2}{\kappa}\hat{\phi}_\kappa \alpha K + \beta^i \partial_i \hat{\phi}_\kappa - \frac{2}{\kappa}\hat{\phi}_\kappa \partial_i \beta^i,$$

$$\partial_t \tilde{\gamma}_{ab} = -2\alpha \tilde{A}_{ab} + \beta^i \partial_i \tilde{\gamma}_{ab} + 2\tilde{\gamma}_{i(a}\partial_{b)}\beta^i - \frac{2}{3}\tilde{\gamma}_{ab}\partial_i \beta^i,$$

$$\partial_t K = -D_i D^i \alpha + \alpha(A_{ij}A^{ij} + \frac{1}{3}K^2) + \beta^i \partial_i K,$$

$$\partial_t \tilde{A}_{ab} = (\hat{\phi}_\kappa)^{\kappa/3}(-D_a D_b \alpha + \alpha R_{ab})^{\mathrm{TF}} + \beta^i \partial_i \tilde{A}_{ab} + 2\tilde{A}_{i(a}\partial_{b)}\beta^i - \frac{2}{3}A_{ab}\partial_i \beta^i,$$

$$\partial_t \tilde{\Gamma}^a = \tilde{\gamma}^{ij}\partial_i \beta_j \beta^a + \frac{1}{3}\tilde{\gamma}^{ai}\partial_i \partial_j \beta^j - \tilde{\Gamma}^i \partial_i \beta^a + \frac{2}{3}\tilde{\Gamma}^a \partial_i \beta^i - 2\tilde{A}^{ai}\partial_i \alpha + 2\alpha(\tilde{\Gamma}^a_{ij}\tilde{A}^{ij} - \frac{\kappa}{2}\tilde{A}^{ai}\frac{\partial_i \hat{\phi}_\kappa}{\hat{\phi}_\kappa} - \frac{2}{3}\tilde{\gamma}^{ai}\partial_i K),$$

$$R_{ij} = \widetilde{R}_{ij} + R^{\phi}_{ij},$$

$$R^{\phi}_{ij} = -2\widetilde{D}_i \widetilde{D}_j \phi - 2\widetilde{\gamma}_{ij}\widetilde{D}^k \widetilde{D}_k \phi + 4\widetilde{D}_i \phi \widetilde{D}_j \phi - 4\widetilde{\gamma}_{ij}\widetilde{D}^k \phi \widetilde{D}_k \phi,$$

$$\widetilde{R}_{ij} = -\frac{1}{2}\widetilde{\gamma}^{lm}\partial_l \partial_m \widetilde{\gamma}_{ij} + \widetilde{\gamma}_{k(i}\partial_{j)}\widetilde{\Gamma}^k + \widetilde{\Gamma}^k \widetilde{\Gamma}_{(ij)k} + \widetilde{\gamma}^{lm}(2\widetilde{\Gamma}^k{}_{l(i}\widetilde{\Gamma}_{j)km} + \widetilde{\Gamma}^k{}_{im}\widetilde{\Gamma}_{klj}).$$

$$\partial_t \alpha - \beta^i \partial_i \alpha = -2\alpha K,$$

$$\partial_t \beta^a - \beta^i \partial_i \beta^a = \frac{3}{4}\alpha B^a,$$

$$\partial_t B^a - \beta^j \partial_j B^i = \partial_t \tilde{\Gamma}^a - \beta^i \partial_i \tilde{\Gamma}^a - \eta B^a,$$

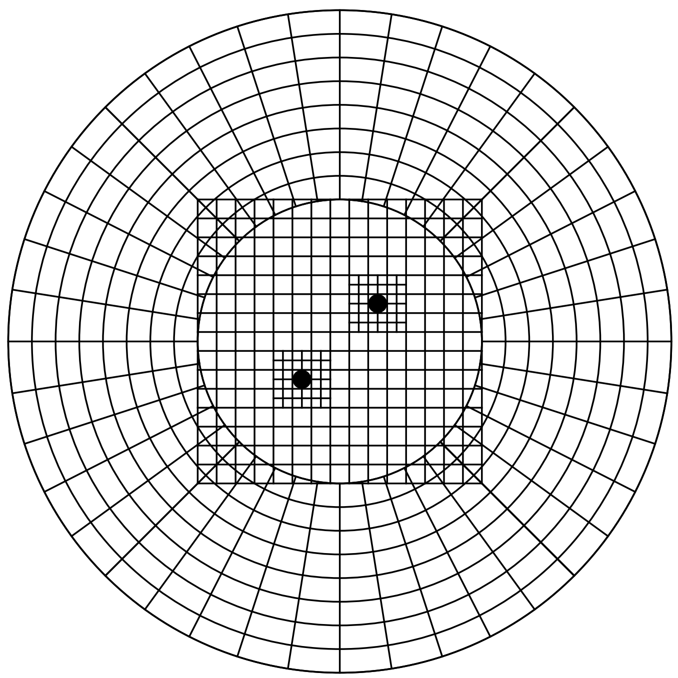
$$\mathcal{H} \equiv R^{(3)} + K^2 - K_{ij}K^{ij} = 0,$$

$$\mathcal{M}^a \equiv D_i(K^{ai} - \gamma^{ai}K) = 0.$$

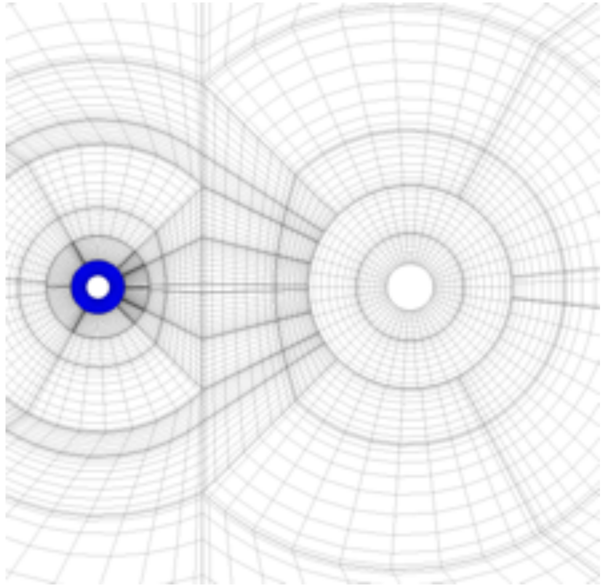
- Now expand components...

Numerical Relativity: Numerics

- Strong field: numerical where all else fails
- State vector of solution on a **3D grid** of points
- Spatial derivatives:
  - High-order **finite differencing**; or
  - **Spectral** collocation
- **Adaptive mesh refinement** in space and time
- **Formal stability**
  - First order in time, **second order** in space; standard methods inapplicable**. Stability** proved for certain formulations *[Calabrese, IH and Husa, 2006]*



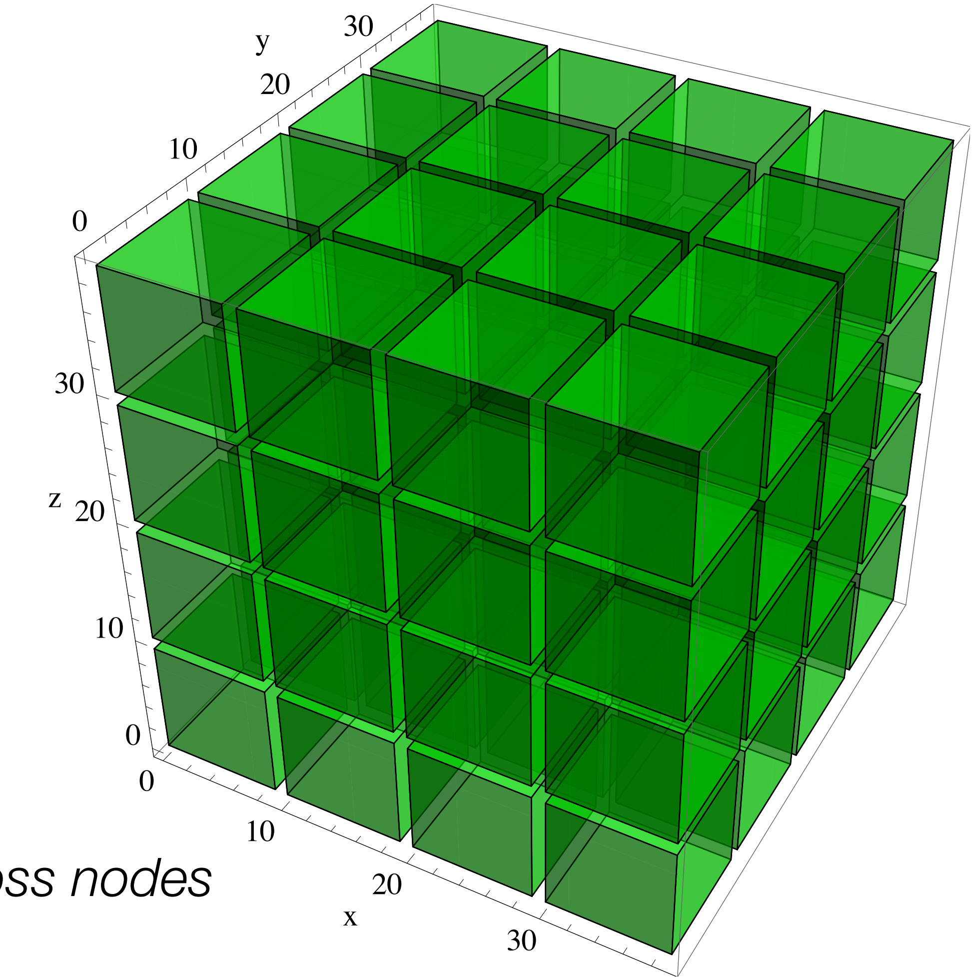
Einstein Toolkit code



SpEC code

Numerical Relativity: Computing

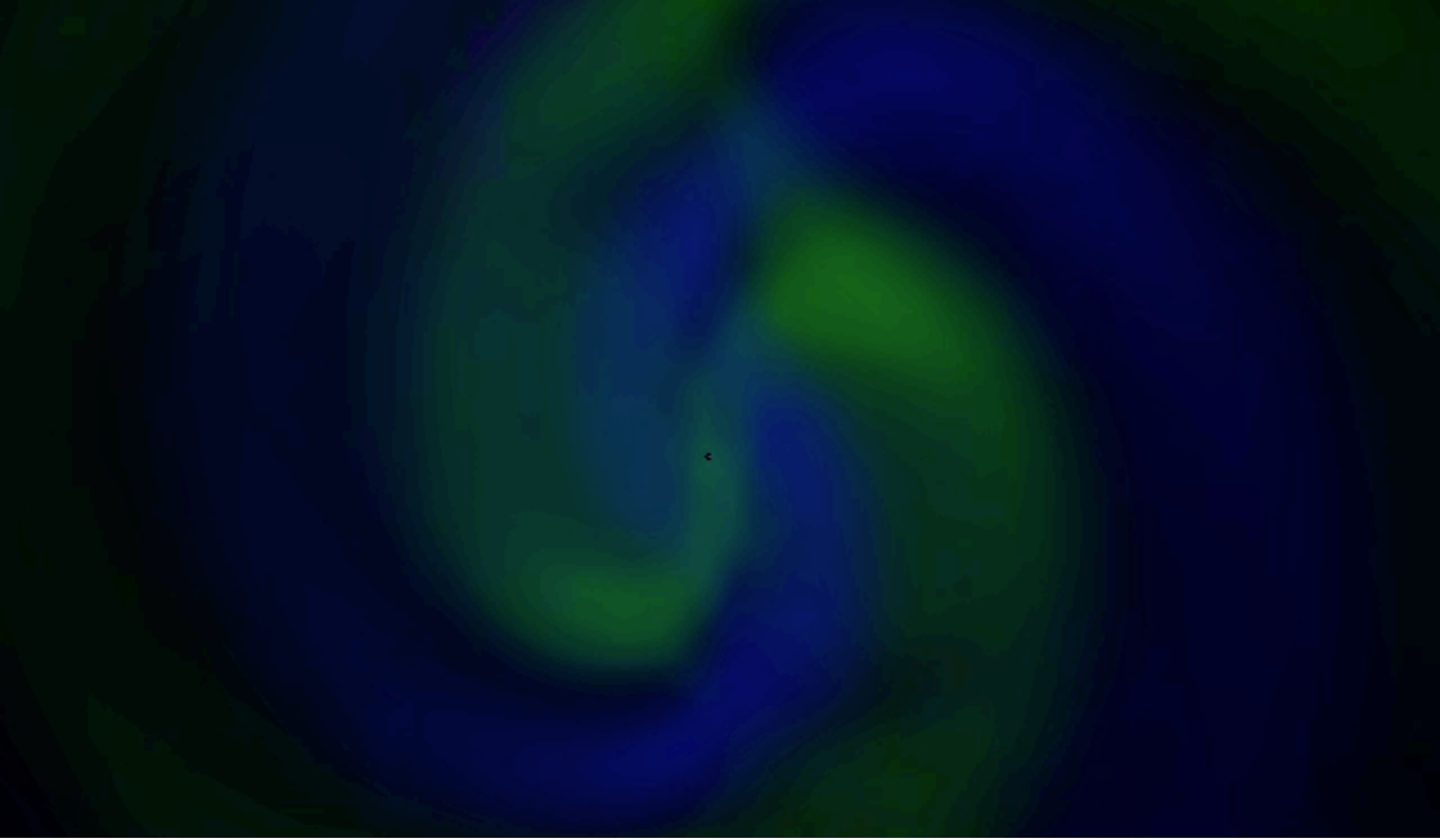
- Simulations: 100 - **1000 cores**, days/weeks/months
- Challenges:
  - **Parallel scalability** (many variables per grid point)
  - Complicated **communication** patterns of AMR
  - Use of modern accelerator architectures:
    - **GPUs**: first BBH simulation *[Blazewicz, IH et al, 2013]*
    - **Intel MIC** systems (Intel "Knights Landing") – work in progress
  - Load imbalance: development of task-based scheduling



Decomposition of a simple numerical domain across nodes

Numerical Relativity: Software

- Einstein Toolkit (einsteintoolkit.org*):*
  - **Open-source** collection of relativity codes *[Löffler, ..., IH et al., 2012]*
  - Based on Cactus, a software **framework** for HPC: portable, established, successful (Gordon Bell prize 2001)
  - **Automated** code generation from tensorial descriptions *[Husa, IH and Lechner, 2004]*
  - **Production**-level, regular releases **tested** on ~30 top-level HPC systems in US and Europe, code review, issue tracking, open mailing list
  - **Funded** by NSF grant #1550551
- **SP**ectral **E**instein **C**ode (SpEC) - black-holes.org
  - Simulating eXtreme Spacetimes (SXS) collaboration
  - Very accurate and efficient
  - **Funded** by Sherman Fairchild Foundation



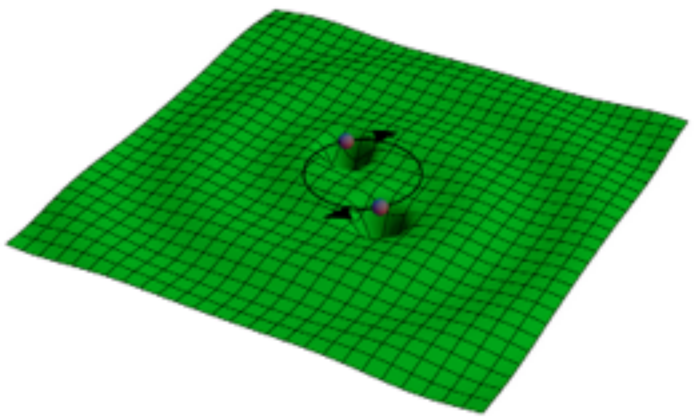
2. Gravitational waves

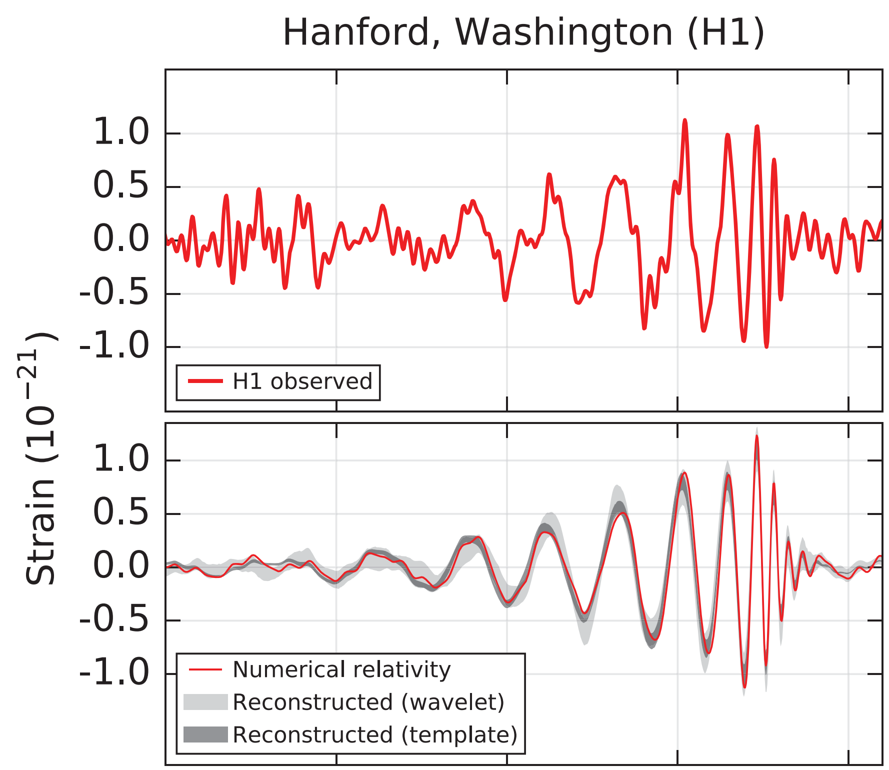
Gravitational wave strain from simulation of GW150914

Understanding gravitational wave signals

- GW detector measures **strain** in TT gauge
- **Matched filtering** to measure signal in **noisy** data
- Parameter estimation:
  - $\langle h_1, h_2 \rangle \equiv 4\,\mathrm{Re} \int_0^\infty \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_h(f)} df$ $\langle \hat{h}_\lambda, \hat{h}_{\mathrm{det}} \rangle$
- Need accurate h_λ:
  - Numerical Relativity: weeks or **months** per waveform, vs **millions** needed
  - Fast models: PN+NR-inspired
- NR: (i) **calibrate** models, (ii) test **systematic** errors, (iii) model-independent **direct** parameter estimation
- In progress: automated NR **pipeline** using the **Einstein Toolkit** (with Huerta and Haas from NCSA) - open science

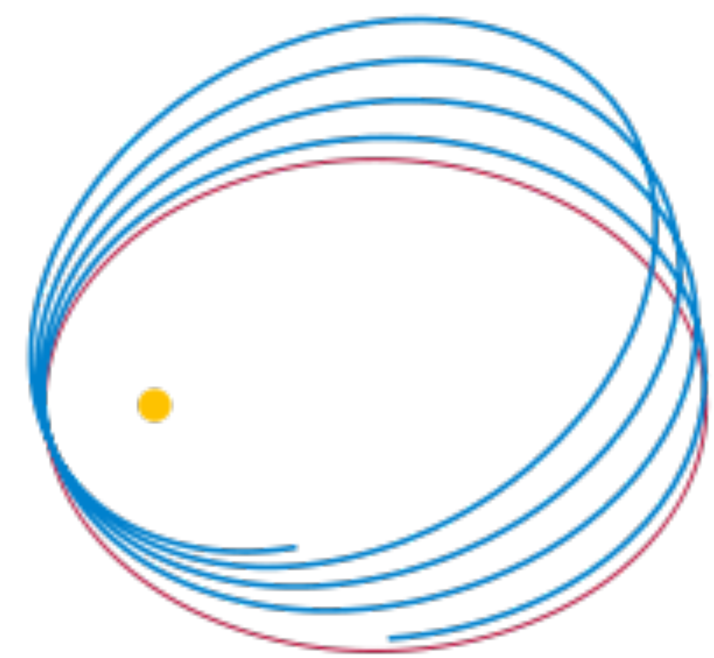
$$h_{ab} = \begin{pmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{pmatrix}$$





The relativistic 2 body problem: Eccentric case

- Eccentric binary systems **circularise** as E and L are emitted *[Peters 1964]*
- LIGO: **circular only**
- Dense stellar environments → non-negligible waveform eccentricity
- Measure/bound eccentricity of **GW events** such as GW150914?
- **Eccentric waveform model:** compare with GW data
  - Use **Post-Newtonian** approximation and **Numerical Relativity**



Post-Newtonian model

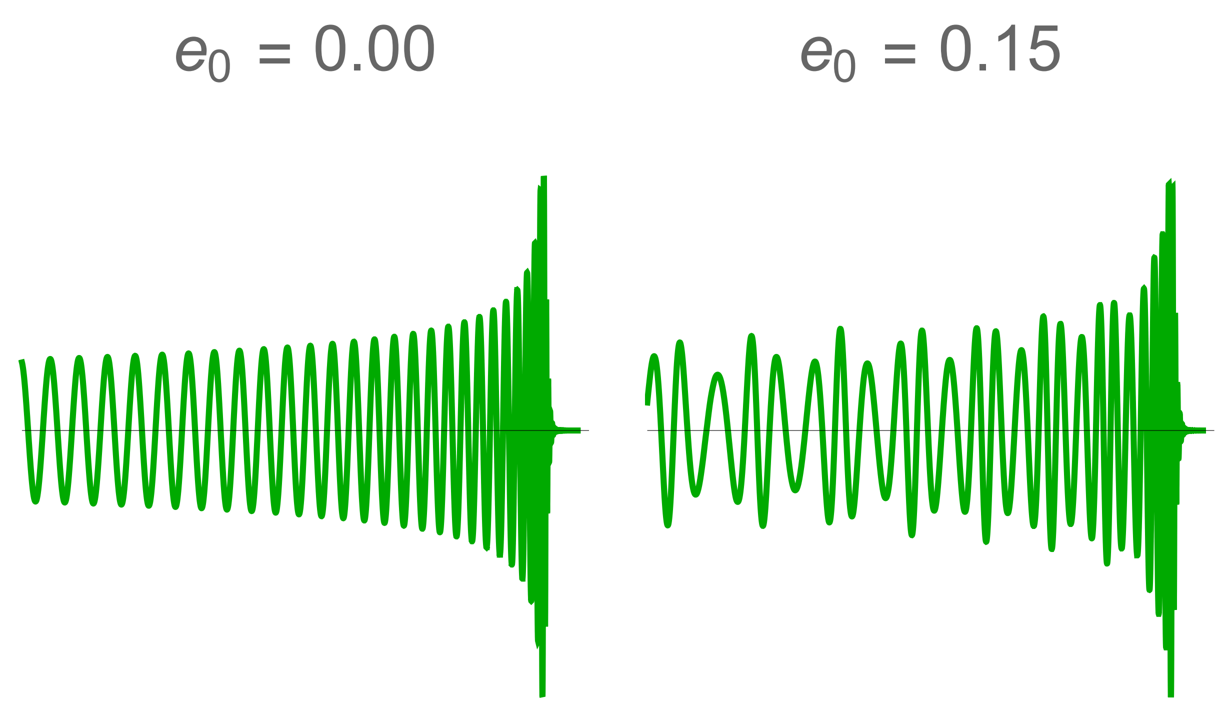
- Large separation: existing **post-Newtonian** approximation:

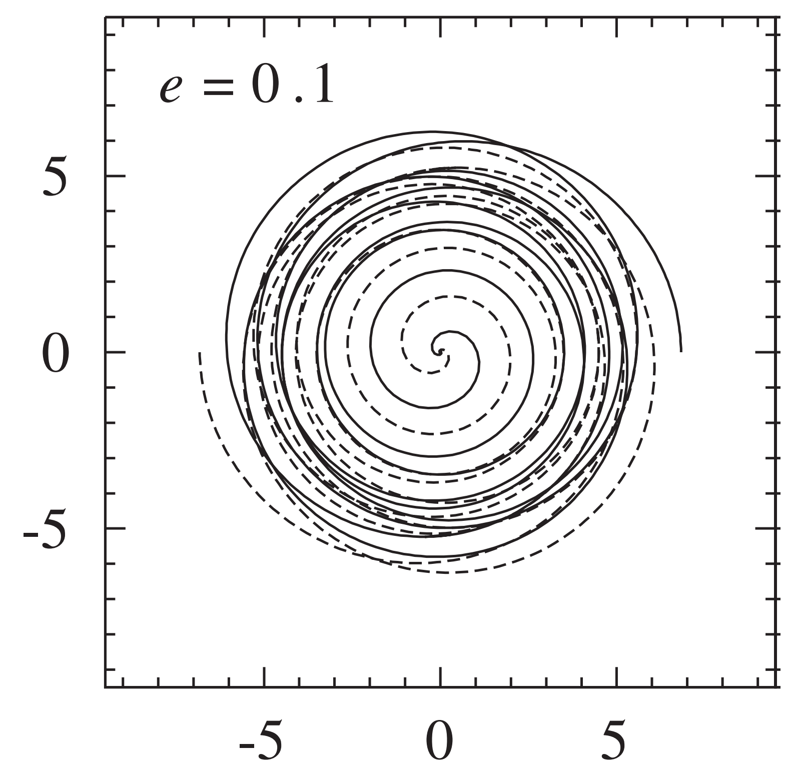
$$\begin{bmatrix} r(t) \\ \phi(t) \end{bmatrix} = \text{expansion in (v/c)}$$

- Breaks down when **v ~ c**
- First **comparison with NR** *[IH et al. 2010]*:
  - Good agreement; depends on **subtleties of PN** model

Numerical Relativity simulations

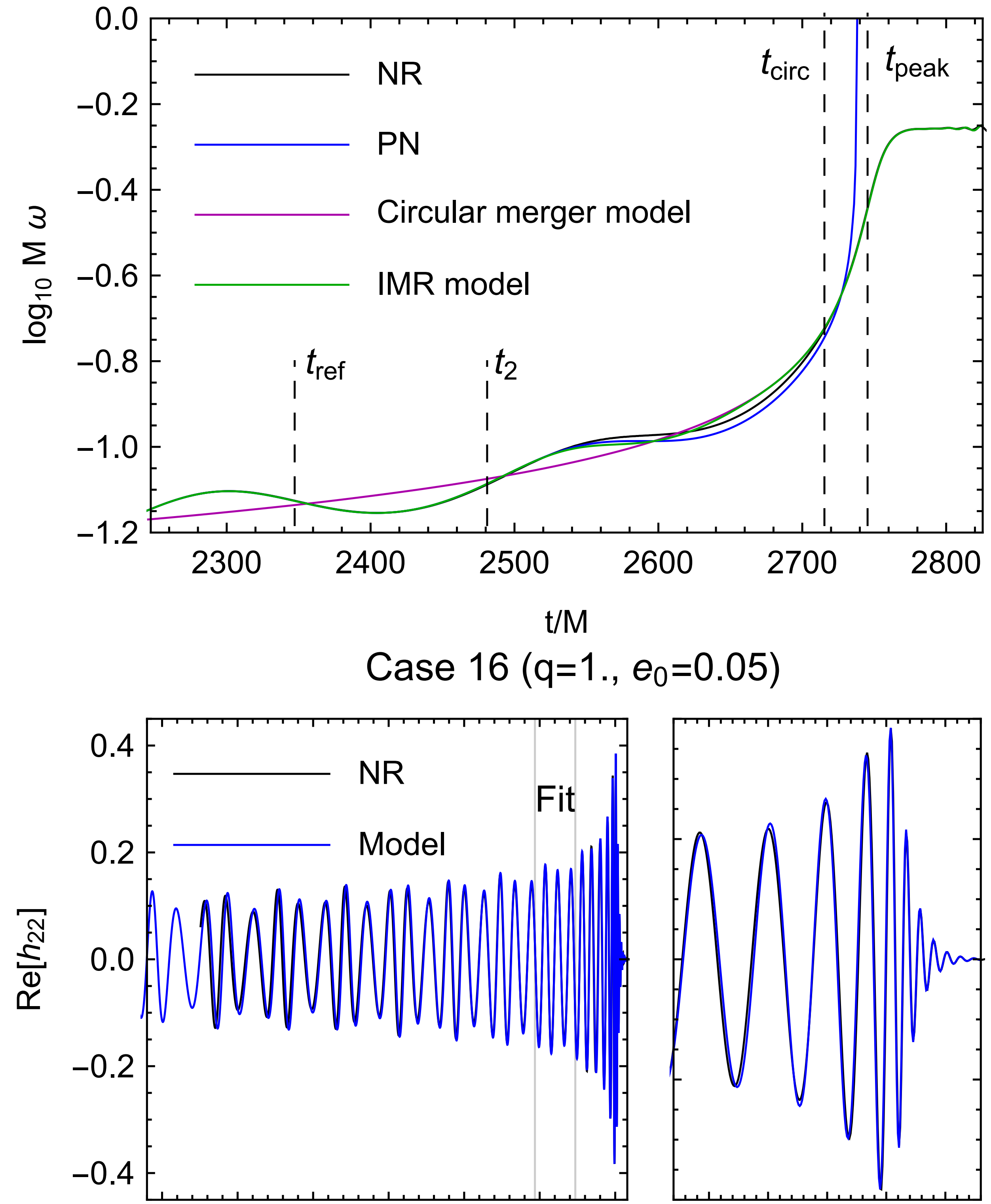
- ~20 new **eccentric NR simulations**
  - ~25 GW cycles with the **SpEC** code
  - Non-spinning
  - Initial eccentricity $e \leq 0.2$
  - $q = m_1/m_2 \leq 3$
- Eccentricity did not bias **GW150914 parameter estimation** *[Pürrer, ..., IH et al 2016]*
- Eccentric binaries **circularise** just before the merger (extending *[IH et al., 2008]*)





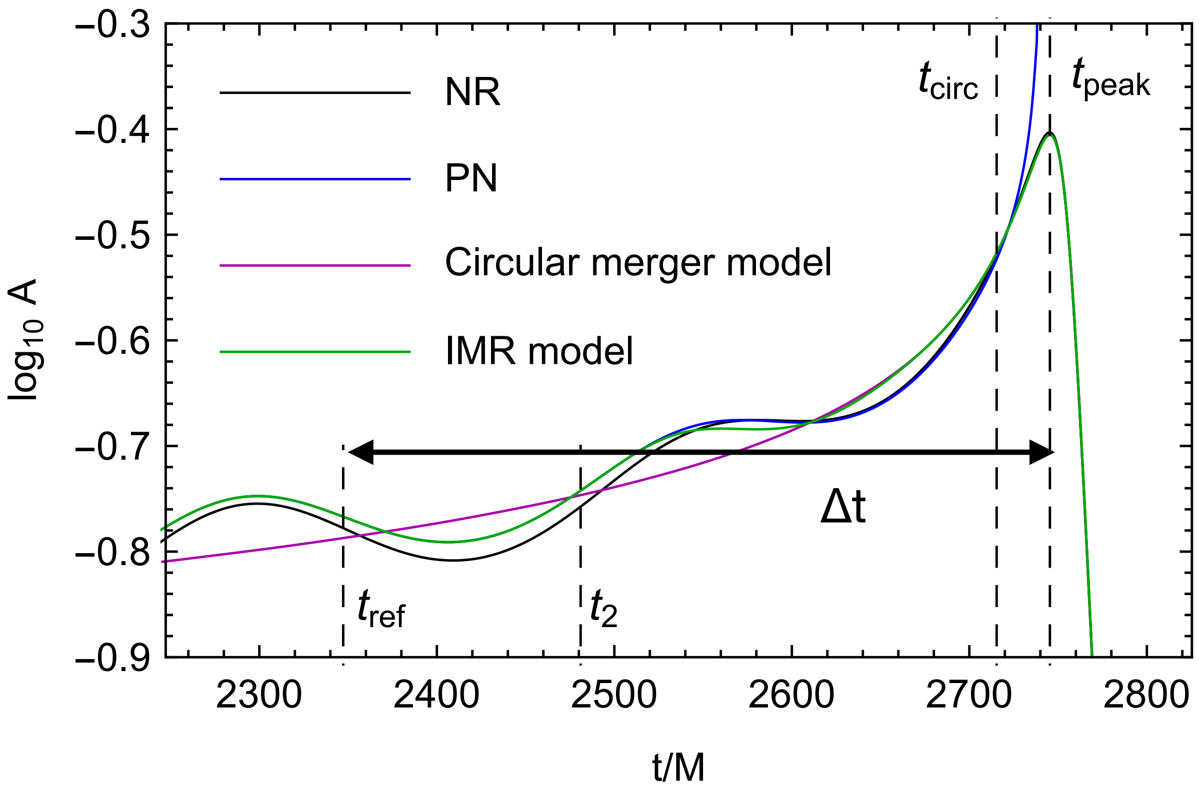
Eccentric waveform model construction

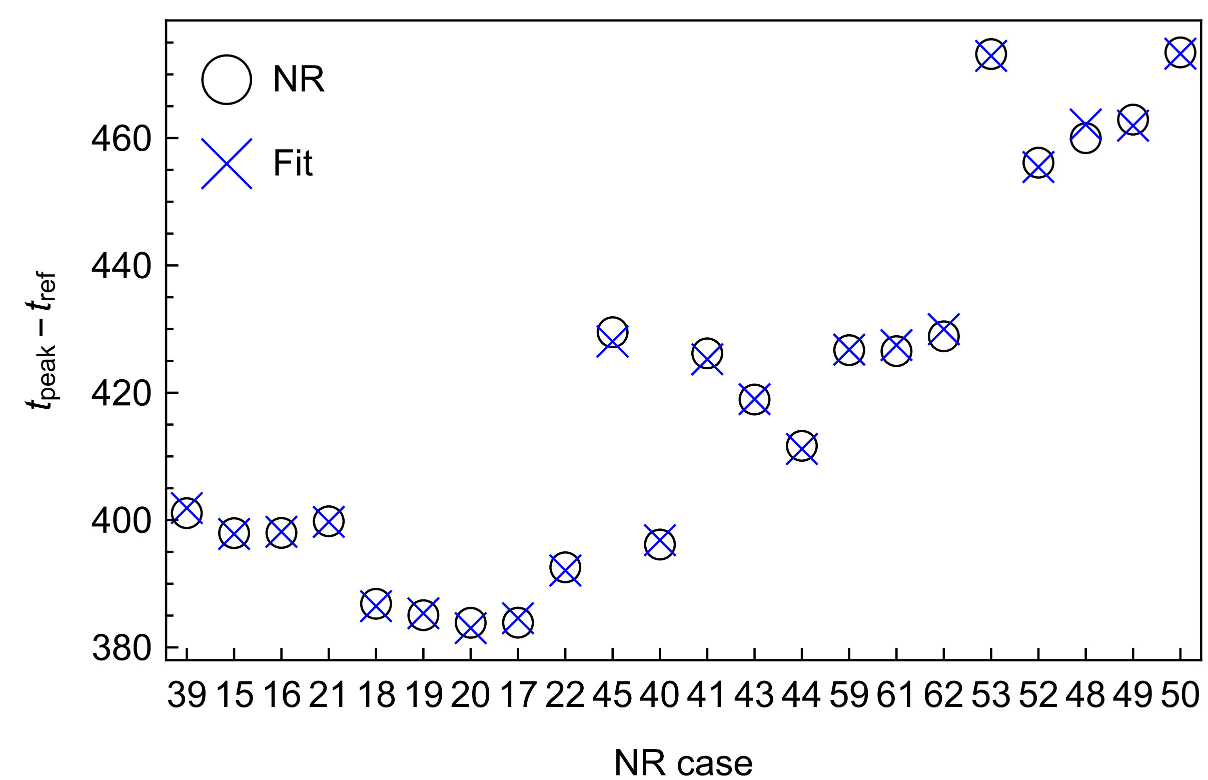
- Inspiral:
  - Use existing eccentric **post-Newtonian** model
- Merger:
  - Use existing **circular** model (justified from observed circularisation)
  - Here, we **interpolate** several equal-mass non-spinning waveforms
- Smoothly **blend** inspiral and merger



Calibration

- **Blending parameters** from NR simulations
- Most important:
  - Δt determining peak of waveform
  - $\Delta t(q, e, l) = \Delta t_0 + a_1 e + a_2 e^2 + b_1 q + b_2 q^2 + c_1 e \cos(l + c_2)$

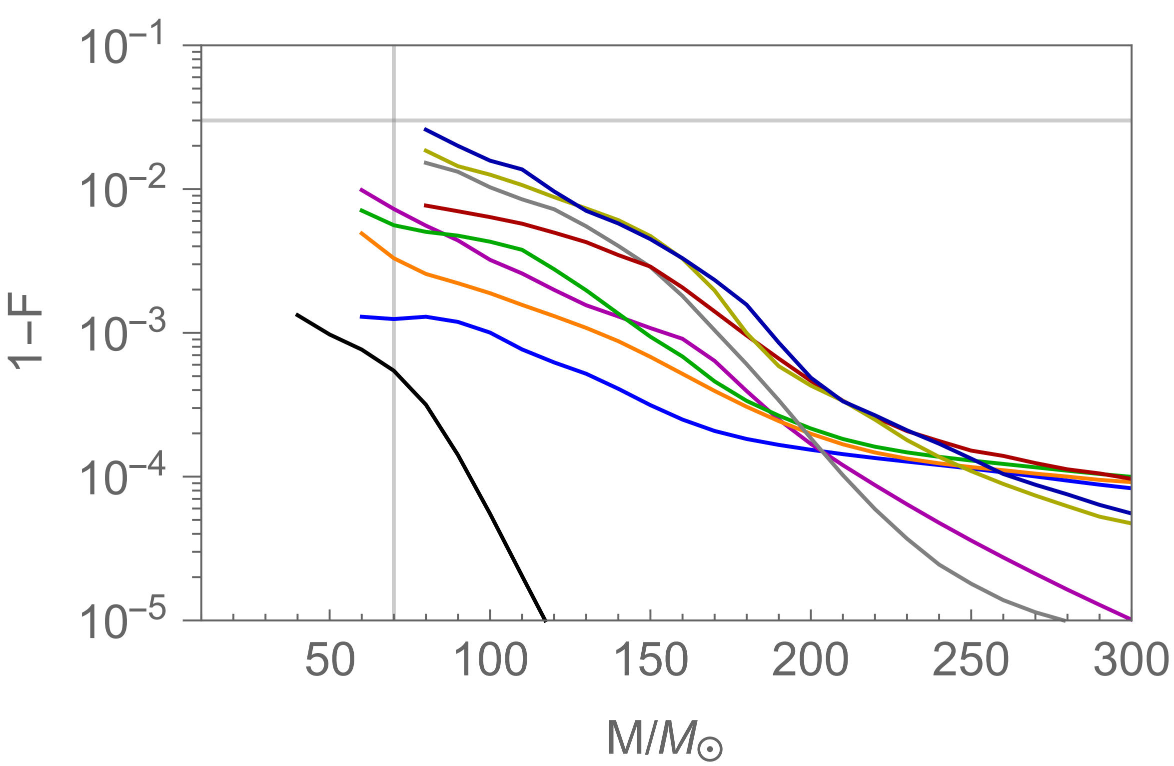




Results

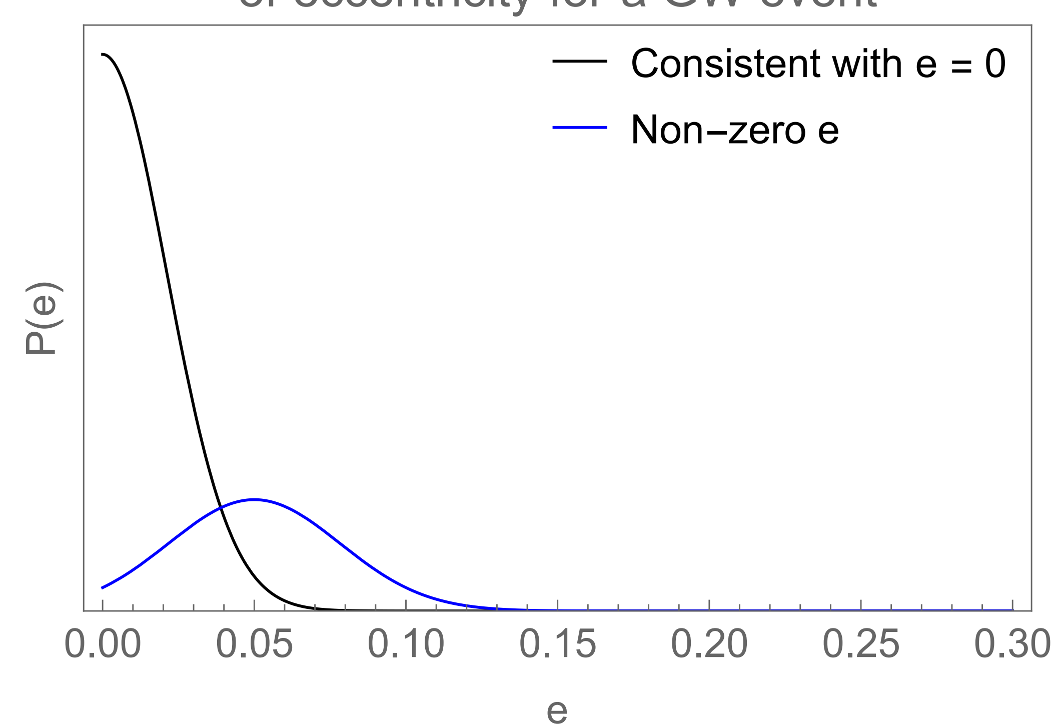
- Now: for configuration like **GW150914,** model **faithfulness** with NR waveform > 97% for systems with
  - Total mass > 93 Msun for e < 0.08
  - Total mass > 70 Msun for e < 0.05

$$F = \langle h_1 | h_2 \rangle = \max_{\phi_c, t_c} \frac{(h_1(\phi_c, t_c) \mid h_2)}{\sqrt{(h_1|h_1)(h_2|h_2)}}$$

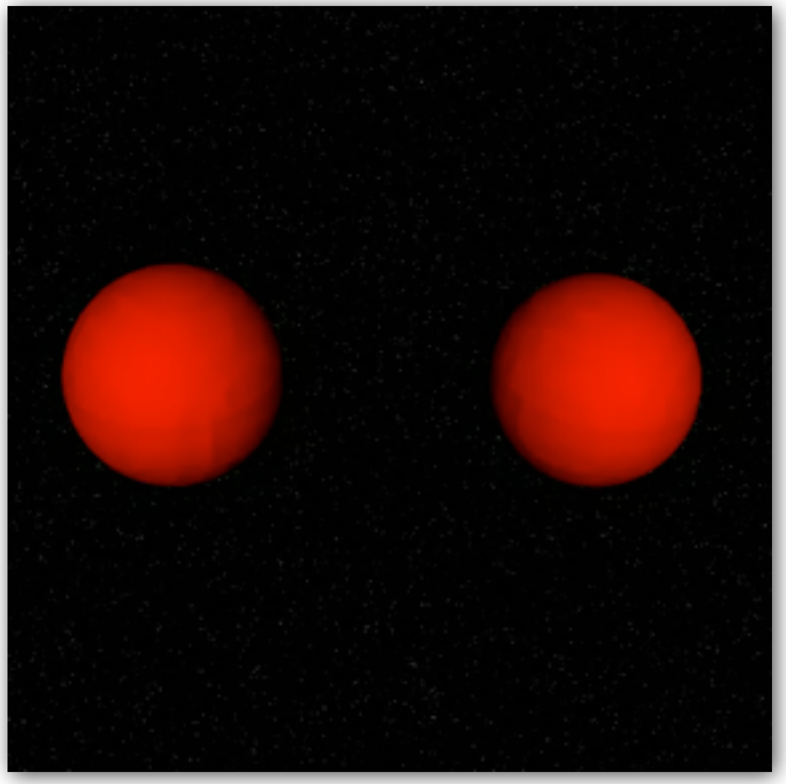


- Future:
  - Include spin
  - Parameter estimation on GW data

Hypothetical parameter estimation of eccentricity for a GW event



Outlook



- NR is **only solution** to Einstein's equations in dynamical strong-field
- LIGO parameter estimation **critically dependent** on NR waveforms
- Soon: **eccentricity** will be measurable
- Some plans...
  - **Eccentric** models for LIGO and LISA with spin
  - Waveforms from high **mass ratio** and high **spin** systems
  - Fully understand numerical **convergence** of NR AMR simulations
  - Design new **NR approximation** methods for **q ~ 100-1000**
  - **Mathematically-rigorous** computation of GWs in NR simulations
  - Task-based **parallelism** and improved **numerical methods** for x10 (?) simulation performance