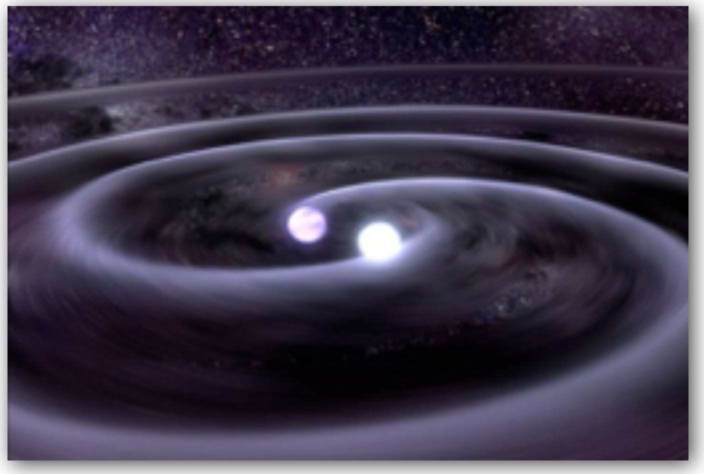
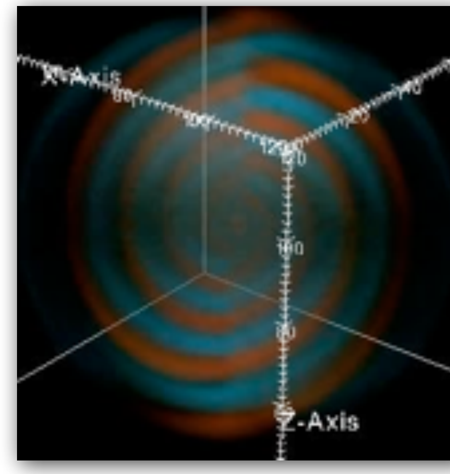




High Performance Computing, Simulation, and Relativity

Ian Hinder

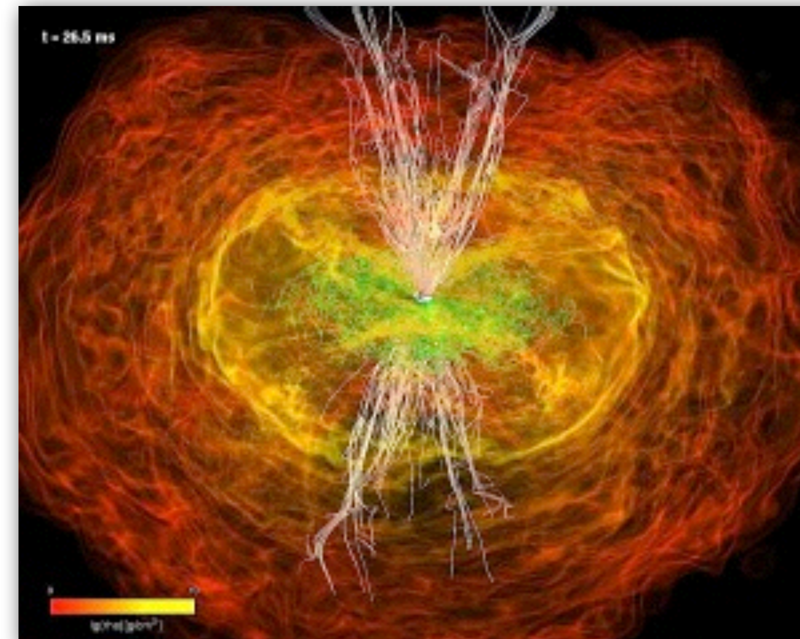
Max Planck Institute for Gravitational Physics,
Potsdam, Germany

10th March 2017
Jürgen Ehlers Spring School, Golm

Too big for a lab? Simulation!

- Can’t **experiment** on black holes/ neutron stars

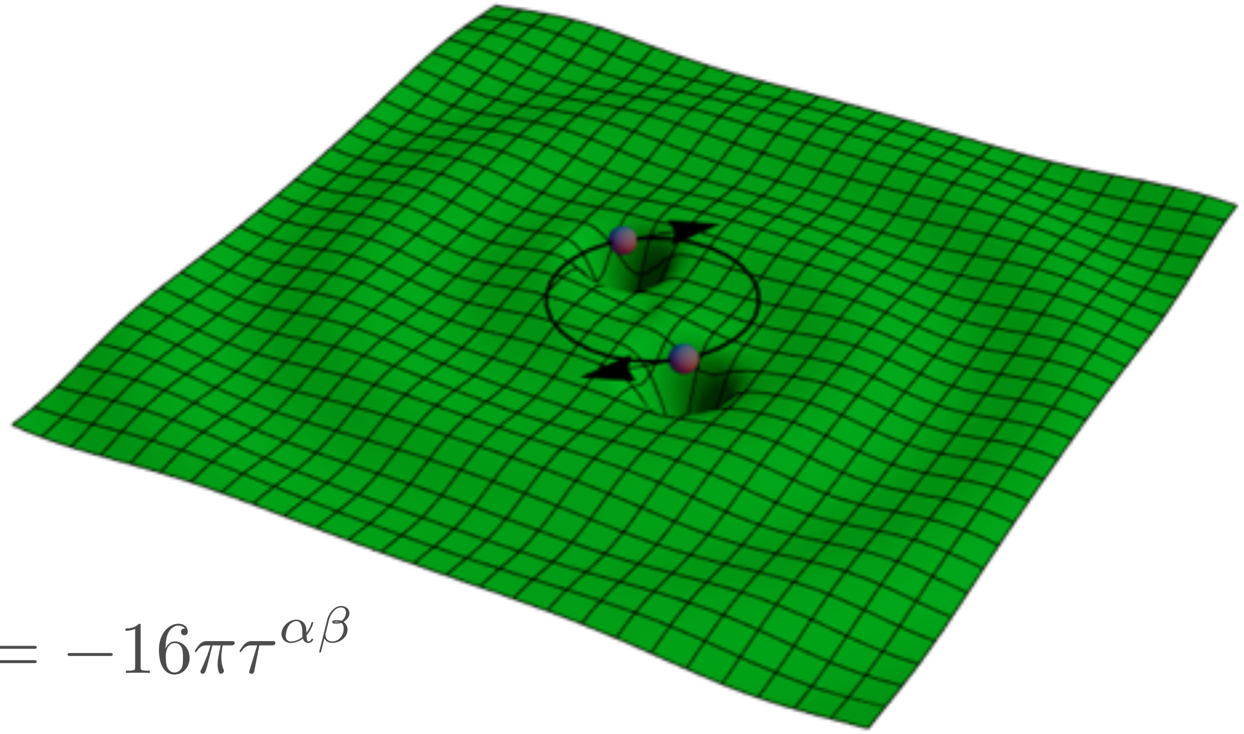
- Use computer **simulations** to see how they behave (assuming certain physics)



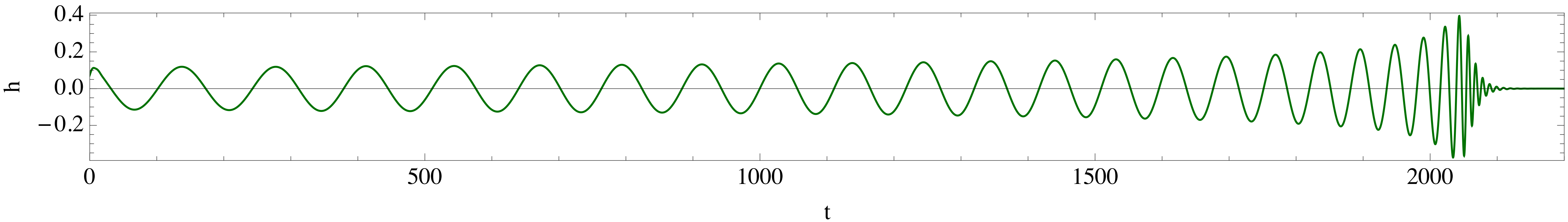
- Compare results with astrophysical **observations**

- Was our **physics model** right?

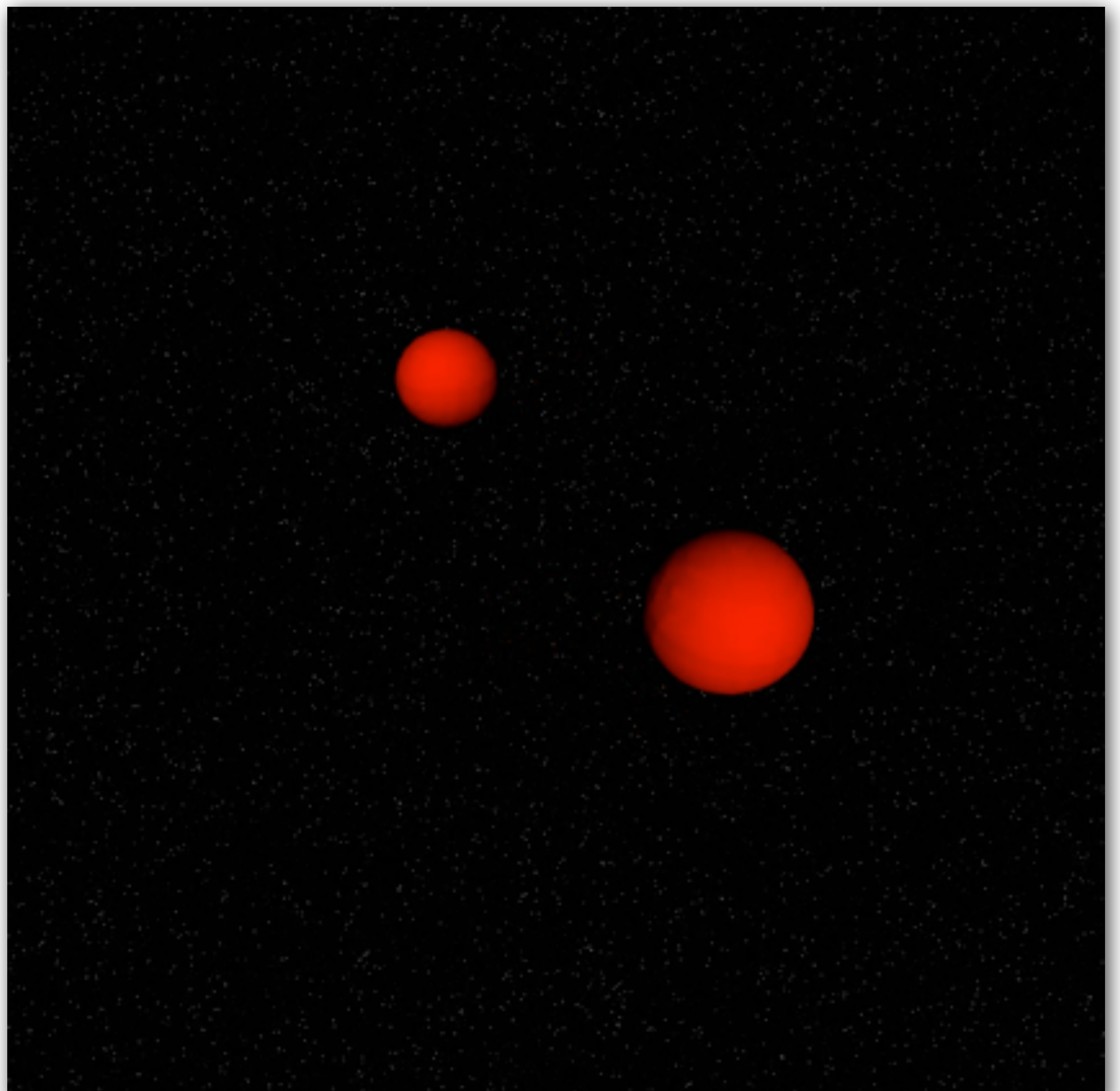
Gravitational waves



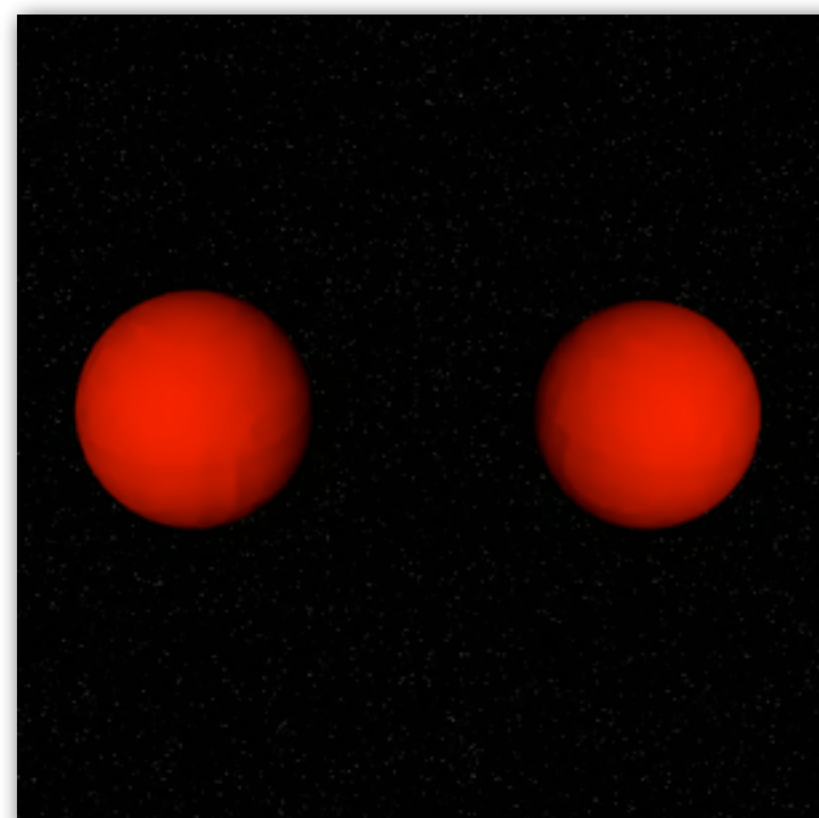
- Dense, fast astrophysical systems can produce **Gravitational Waves**
- First observed in **2015** $(-\partial_t^2 + \nabla^2)h^{\alpha\beta} = -16\pi\tau^{\alpha\beta}$
- Gravitational Wave detectors: **Advanced LIGO/VIRGO**
- **Very weak** signals; need to know what to look for!
- Need **Numerical Relativity** simulations to model the dynamics and predict the waves



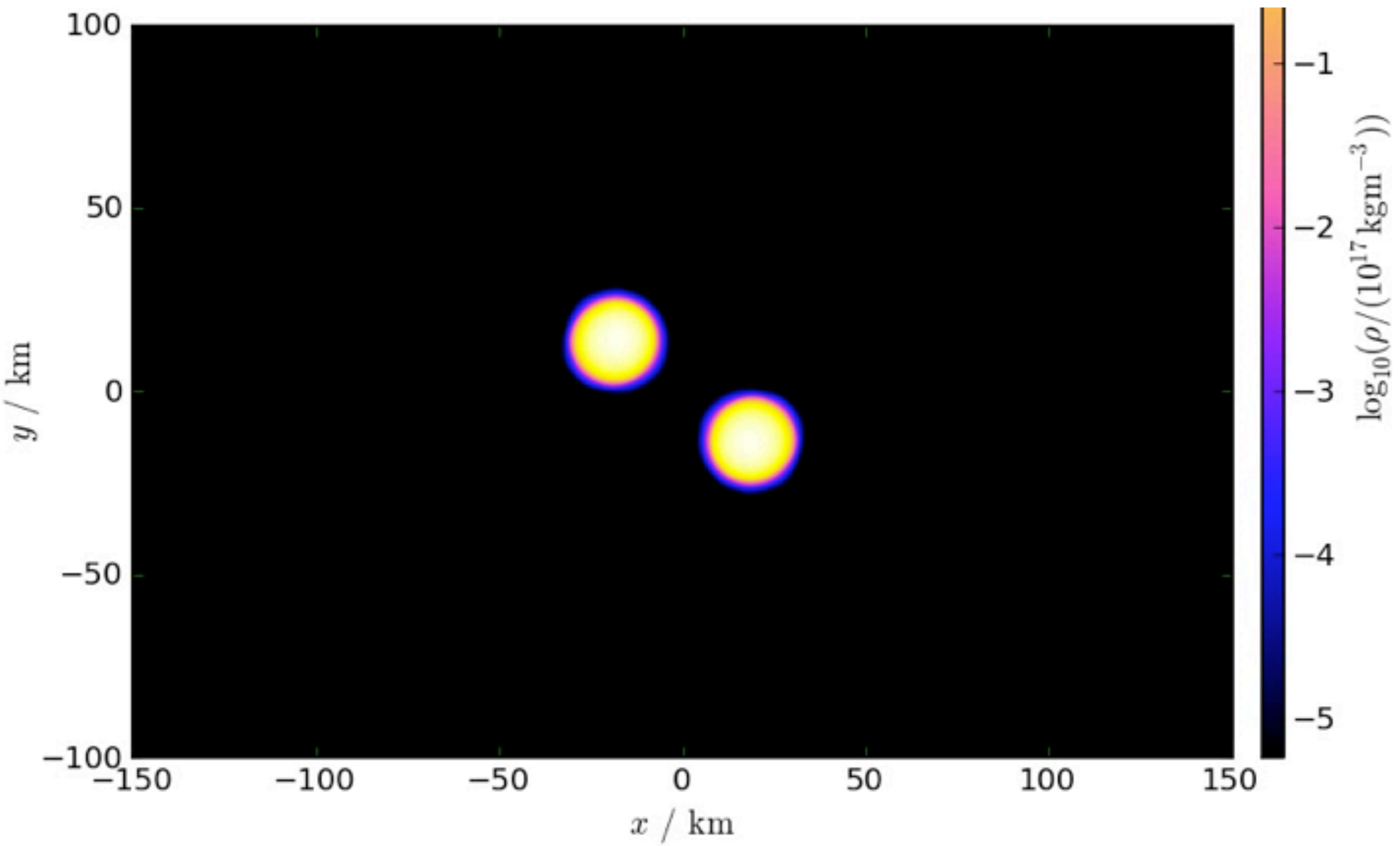
Inspiral and merger of **black hole** binary system



- Black holes **orbiting** around each other
- Lose potential energy by emission of **Gravitational Waves**
- Separation shrinks: black holes **merge**

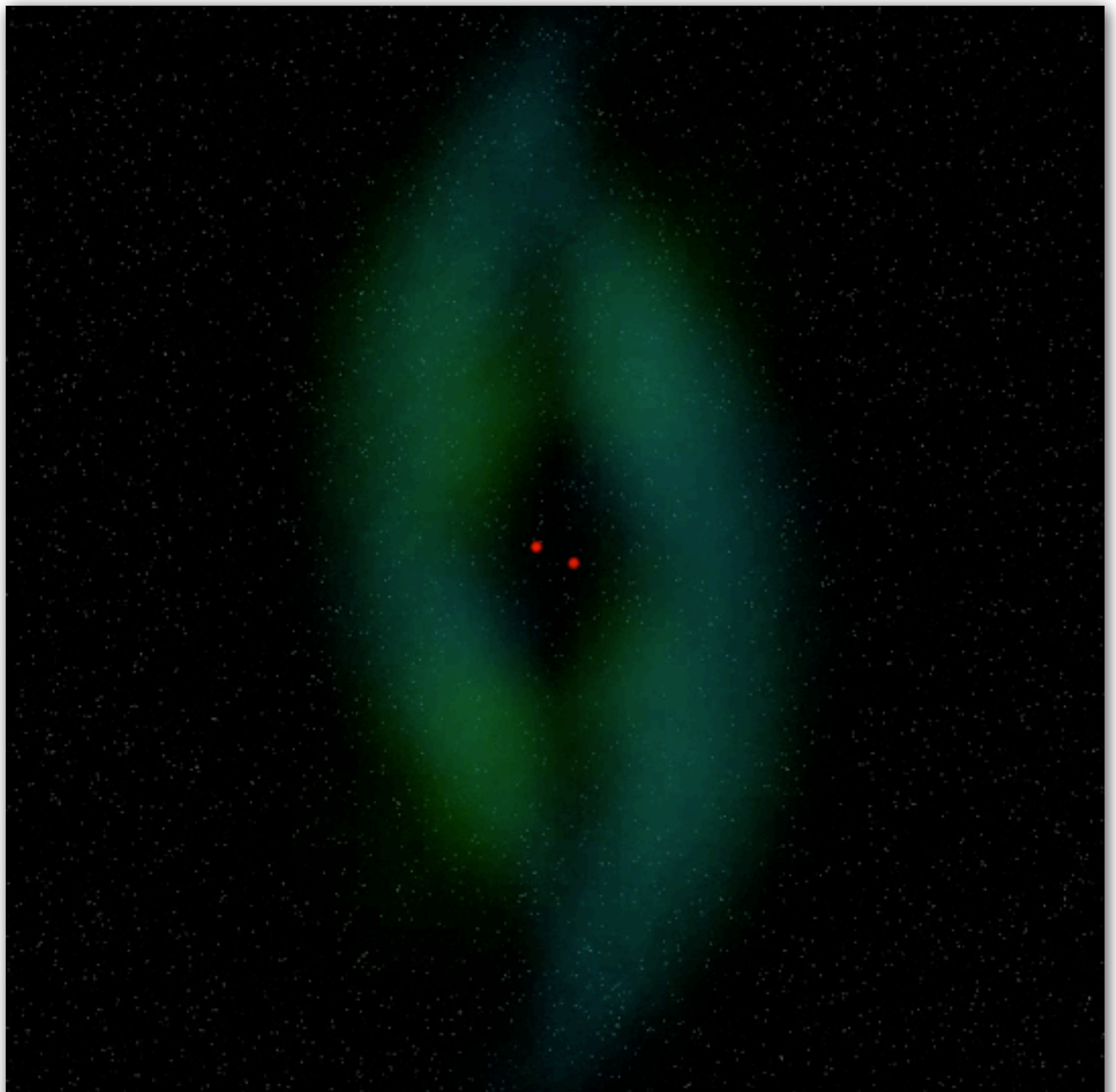


Inspiral and merger of **neutron star** binary system

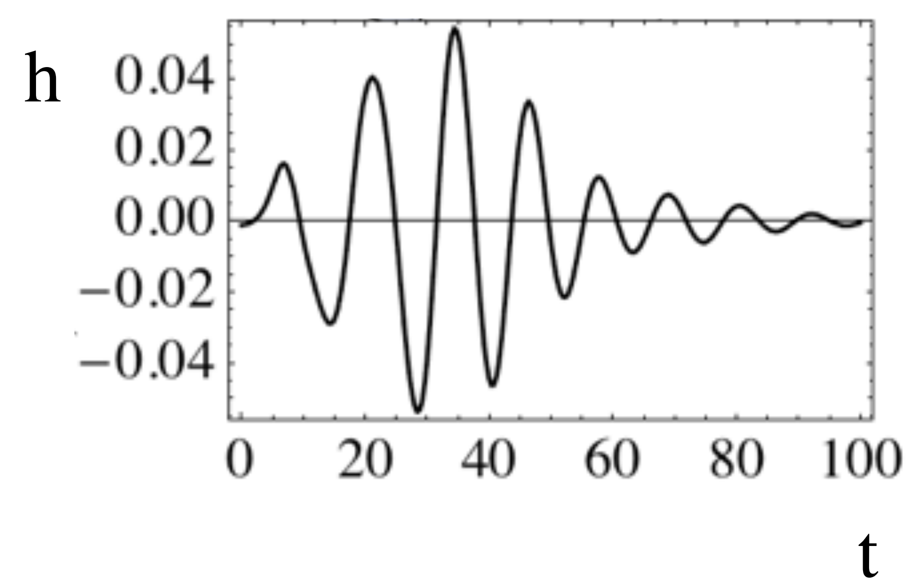


- Stars throw off **matter** as they merge to form a **black hole**
- Matter forms a **disk** in orbit

Gravitational waves from a black hole binary

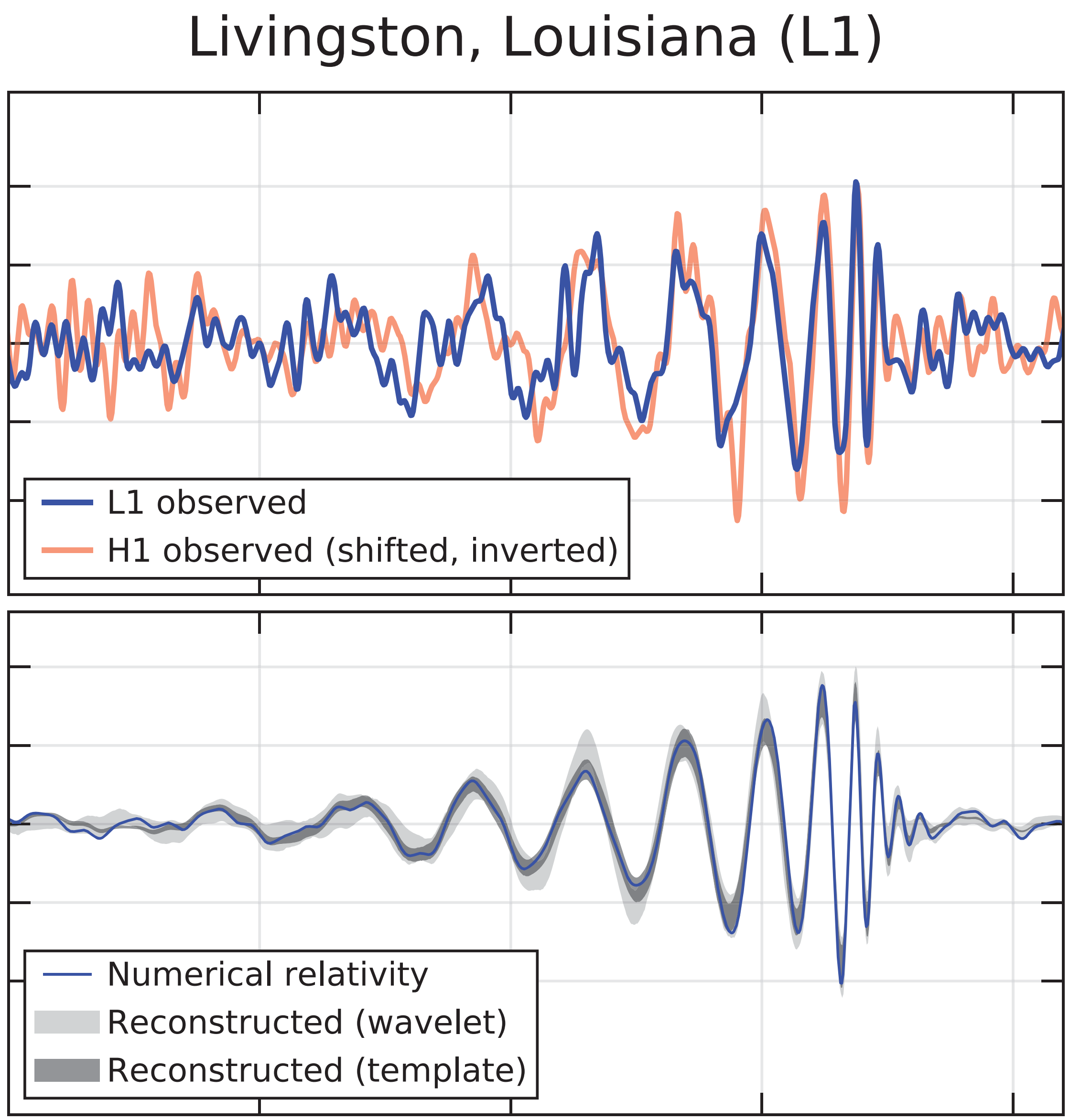


- Gravitational wave strain
  - $h_{\mu\nu}(t, r, \theta, \Phi)$
- Detector measures at a fixed (r, θ, Φ) as a function of time:



GW150914: Observation vs simulation

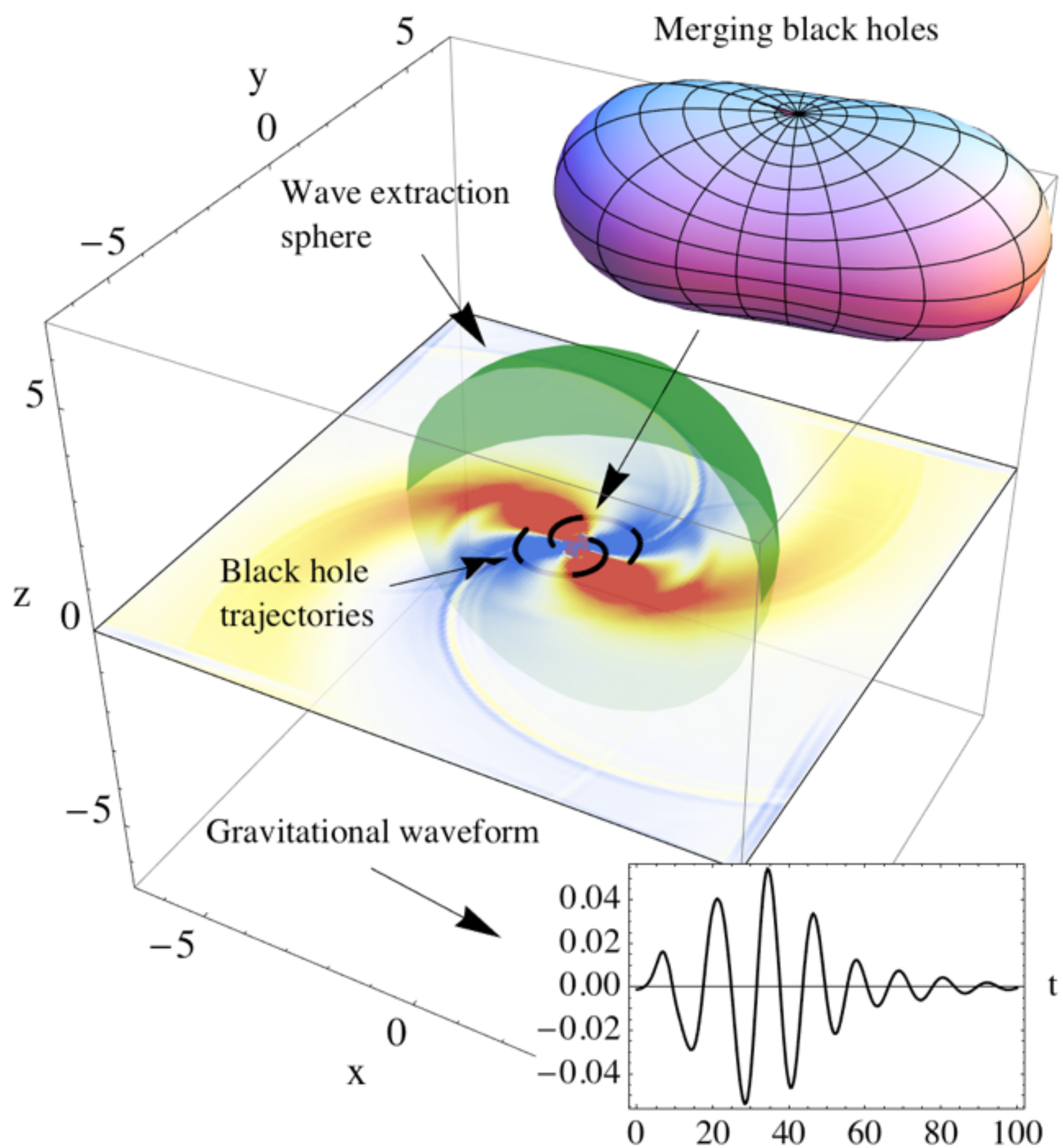
- September 2015: First direct **detection** of gravitational waves (LIGO)
- Excellent **agreement** between observed signal and Numerical Relativity simulations
- In general, require Numerical Relativity to infer **properties** (masses, spins, etc)



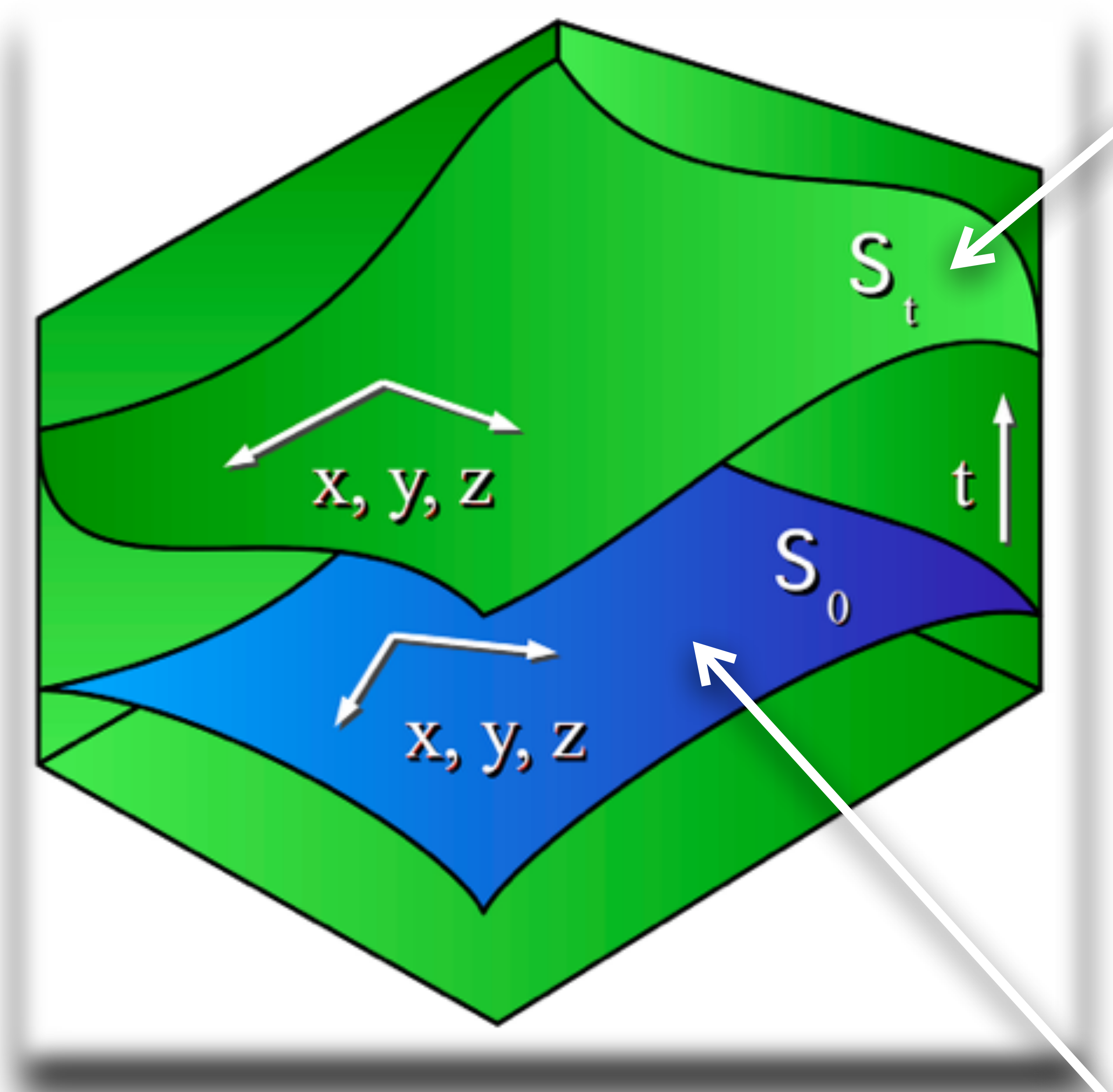
Abbott et al. 2015

Numerical Relativity

- Direct solution of **Einstein's equations** on supercomputers
- Major applications:
  - Binary **black holes** and binary **neutron stars**
  - **Supernova** core collapse
- Size: 100 - **1000 cores**
- Simulation time: days/weeks/months



Einstein’s Equations



Solution at time t

Einstein’s equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Initial data (t = 0)

Einstein equations in time-evolution form

$$\partial_t\hat{\phi}_\kappa = \frac{2}{\kappa}\hat{\phi}_\kappa \alpha K + \beta^i\partial_i\hat{\phi}_\kappa - \frac{2}{\kappa}\hat{\phi}_\kappa\partial_i\beta^i,$$

$$\partial_t\tilde{\gamma}_{ab} = -2\alpha\tilde{A}_{ab} + \beta^i\partial_i\tilde{\gamma}_{ab} + 2\tilde{\gamma}_{i(a}\partial_{b)}\beta^i - \frac{2}{3}\tilde{\gamma}_{ab}\partial_i\beta^i,$$

$$\partial_t K = -D_iD^i\alpha + \alpha(A_{ij}A^{ij} + \frac{1}{3}K^2) + \beta^i\partial_i K,$$

$$\partial_t\tilde{A}_{ab} = (\hat{\phi}_\kappa)^{\kappa/3}(-D_aD_b\alpha + \alpha R_{ab})^{\mathrm{TF}} + \beta^i\partial_i\tilde{A}_{ab} + 2\tilde{A}_{i(a}\partial_{b)}\beta^i - \frac{2}{3}A_{ab}\partial_i\beta^i,$$

$$\partial_t\tilde{\Gamma}^a = \tilde{\gamma}^{ij}\partial_i\beta_j\beta^a + \frac{1}{3}\tilde{\gamma}^{ai}\partial_i\partial_j\beta^j - \tilde{\Gamma}^i\partial_i\beta^a + \frac{2}{3}\tilde{\Gamma}^a\partial_i\beta^i - 2\tilde{A}^{ai}\partial_i\alpha + 2\alpha(\tilde{\Gamma}^a_{ij}\tilde{A}^{ij} - \frac{\kappa}{2}\tilde{A}^{ai}\frac{\partial_i\hat{\phi}_\kappa}{\hat{\phi}_\kappa} - \frac{2}{3}\tilde{\gamma}^{ai}\partial_i K),$$

$$R_{ij} = \widetilde{R}_{ij} + R^{\phi}_{ij},$$

$$R^{\phi}_{ij} = -2\widetilde{D}_i\widetilde{D}_j\phi - 2\widetilde{\gamma}_{ij}\widetilde{D}^k\widetilde{D}_k\phi + 4\widetilde{D}_i\phi\widetilde{D}_j\phi - 4\widetilde{\gamma}_{ij}\widetilde{D}^k\phi\widetilde{D}_k\phi,$$

$$\widetilde{R}_{ij} = -\frac{1}{2}\widetilde{\gamma}^{lm}\partial_l\partial_m\widetilde{\gamma}_{ij} + \widetilde{\gamma}_{k(i}\partial_{j)}\widetilde{\Gamma}^k + \widetilde{\Gamma}^k\widetilde{\Gamma}_{(ij)k} + \widetilde{\gamma}^{lm}(2\widetilde{\Gamma}^k{}_{l(i}\widetilde{\Gamma}_{j)km} + \widetilde{\Gamma}^k{}_{im}\widetilde{\Gamma}_{klj}).$$

$$\partial_t\alpha - \beta^i\partial_i\alpha = -2\alpha K,$$

$$\partial_t\beta^a - \beta^i\partial_i\beta^a = \frac{3}{4}\alpha B^a,$$

$$\partial_t B^a - \beta^j\partial_j B^i = \partial_t\tilde{\Gamma}^a - \beta^i\partial_i\tilde{\Gamma}^a - \eta B^a,$$

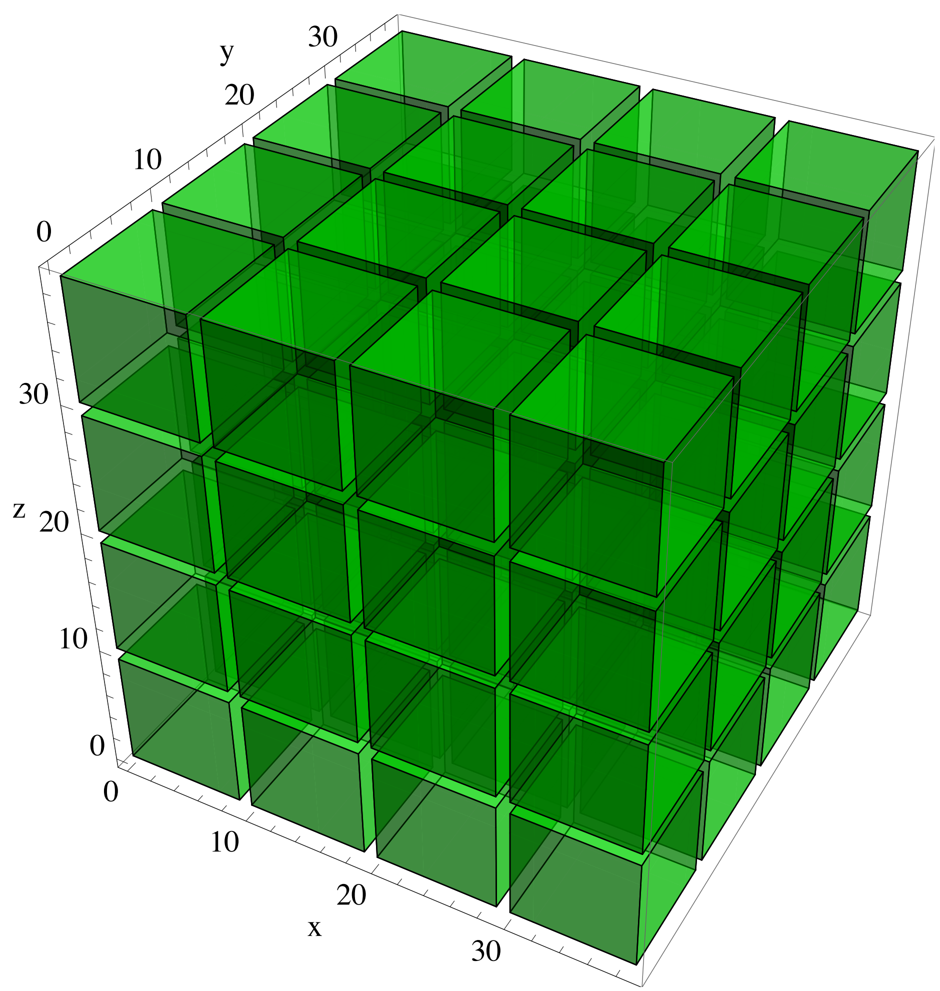
$$\mathcal{H} \equiv R^{(3)} + K^2 - K_{ij}K^{ij} = 0,$$

$$\mathcal{M}^a \equiv D_i(K^{ai} - \gamma^{ai}K) = 0.$$

- **Tensor** equations
- Use **computer algebra** to manipulate/optimise
- Automatically generate **C code** to solve them (15000 lines)

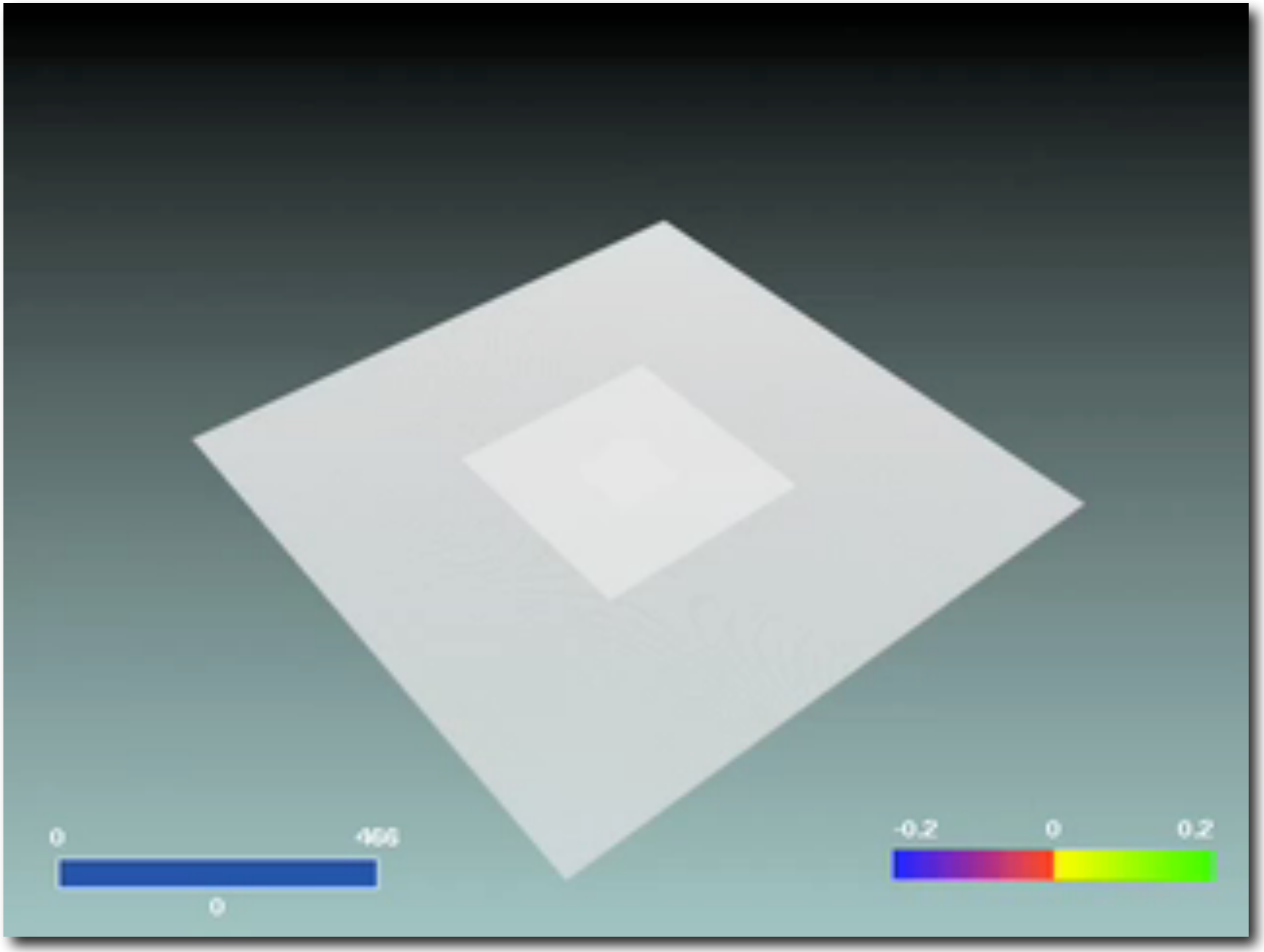
Why supercomputers?

- Need to **store** at least one 3D t = const grid of data **in memory**
- **Too many points** and too many variables to fit in a workstation



- Supercomputer consists of many individual "**nodes**" connected by a fast "**interconnect**" network
- **Split up** the grid into blocks and run each on a separate node
- **Parallel programming** required!

Adaptive Mesh Refinement



The supercomputer in the basement: **Minerva**

- 38 TB of main memory
- 594 nodes (9504 cores)
- 302.4 TFLOPS (3×10^{14} calculations per second)
- 58 Gb/sec communication network
- 500 TB of disk space
- Used for Numerical Relativity: binary black hole and neutron star simulations