

# An eccentric binary black hole inspiral-merger-ringdown gravitational waveform model from post-Newtonian and numerical relativity

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Simulating  
eXtreme  
Spacetimes



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# Introduction

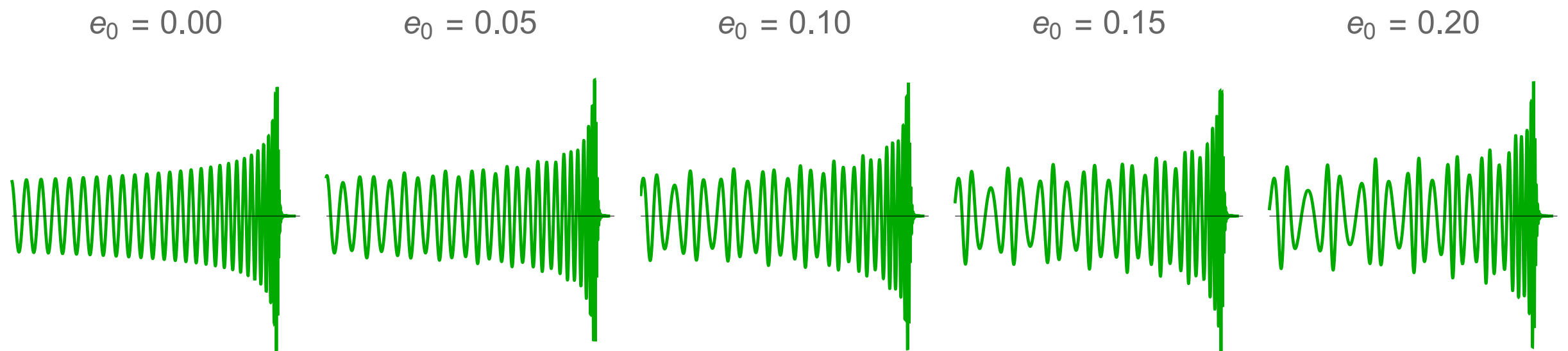
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- Eccentric binary systems **circularise** as E and L are emitted (Peters 1964)
- Eccentricity of BBH **expected** to be 0 well before merger
- Can we measure (bound) eccentricity of **GW events** such as GW150914?
- **Eccentric waveform model** could be compared with GW data to measure/constrain eccentricity
- Construct and test such a model using **Post-Newtonian** approximation and **Numerical Relativity**
- Only need late inspiral+merger; e.g. **last 5 orbits** for GW150914

# A selection of eccentric NR simulations

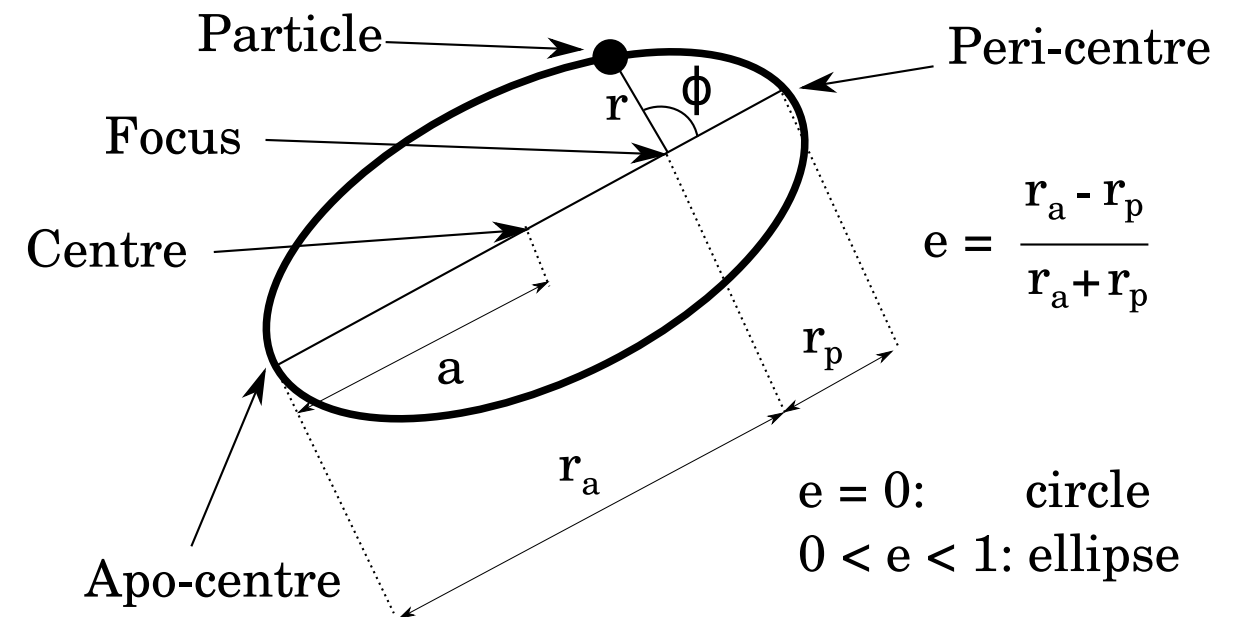
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- ~12 orbits with the **SpEC** code
- Non-spinning
- Initial eccentricity  **$e \leq 0.2$**
- $q = m_1/m_2 \leq 3$



# Modelling the inspiral: the building blocks

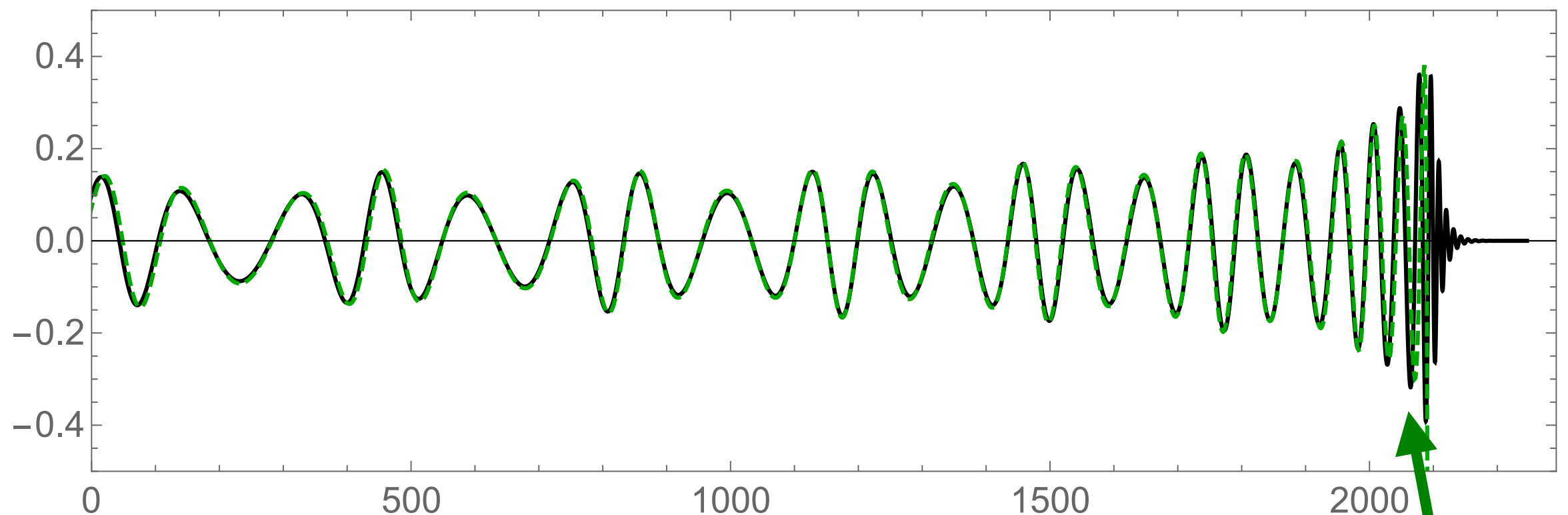
- **Post-Newtonian** model:
  - **Conservative** motion (without inspiral):
    - constant E and L
    - eccentricity **e**, semi-major axis **a**
    - $r, \phi$  in E and L (**3 PN**)
  - **Radiation reaction**:
    - Adiabatic constants E and L integrated from 2 **PN fluxes**
  - **Waveforms** 0 PN (restricted approximation):
    - $h_+, h_x$  in  $r, \phi$



- See Hinder et al. 2010 for details
- Empirically found best agreement with NR for PN expansion variable  $x$  (TaylorT4  $x$  when  $e \rightarrow 0$ )

# Validation of PN inspiral against Numerical Relativity

- NR and PN agree well in inspiral for last  $\sim 10$  orbits



- Fit** best PN ( $e, x, l, \phi$ ) over inspiral

PN breaks  
down near  
merger

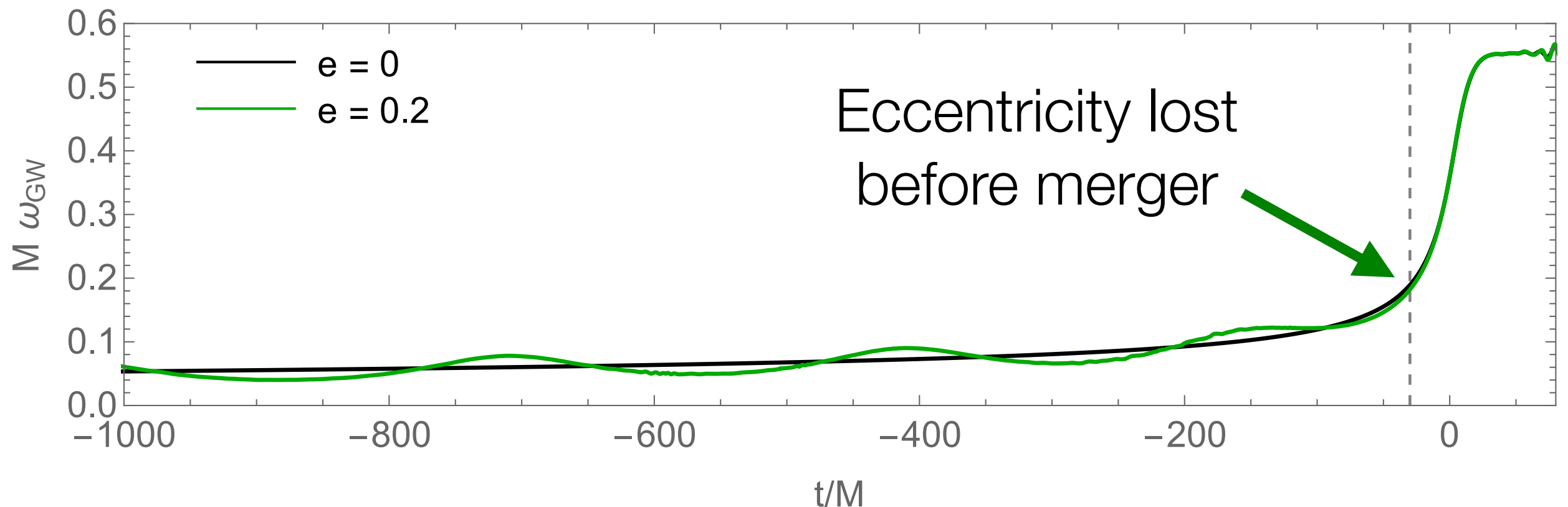
# How to model the merger?

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- Use an **effective model** based on physical insight
  - Likely to generalise outside calibration parameter space
  - See talk by **Eliu Huerta** on Wednesday in C2 GW session
- Fitting to **NR simulations**
  - Sufficient if NR parameter space **covers region of interest**

# What does an eccentric BBH merger look like?

- Eccentric mergers are **circular** (Hinder et al. 2008)

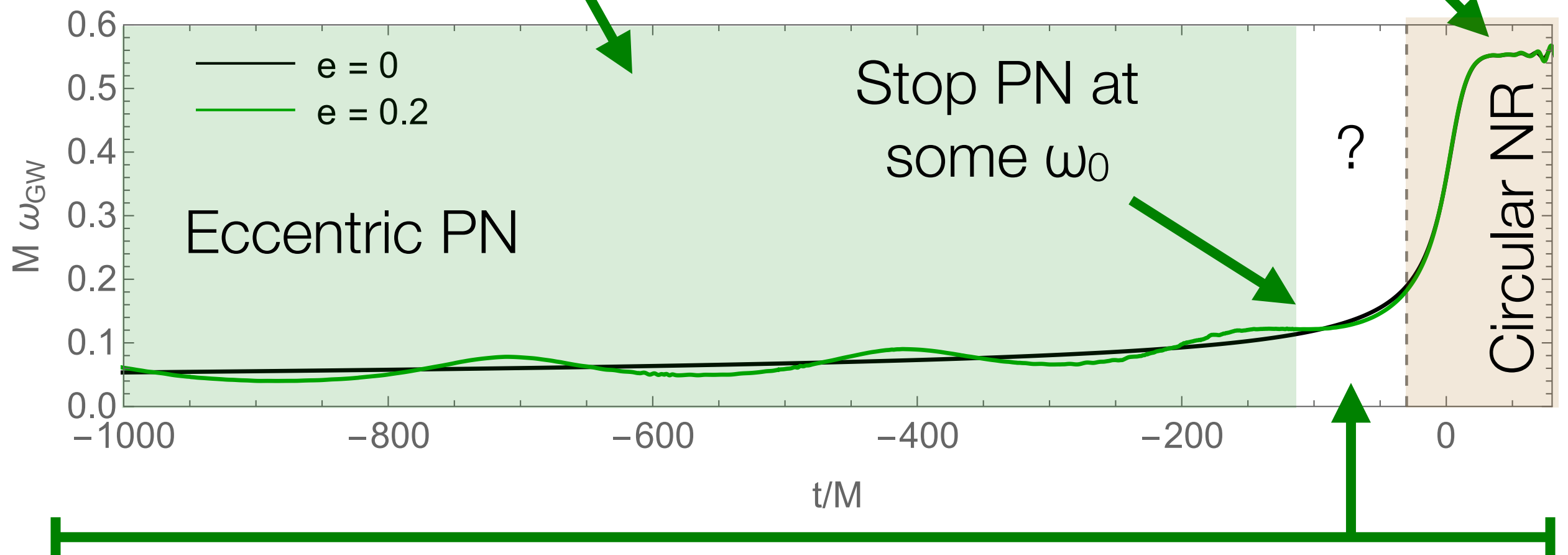


- Circularisation in **frequency** and **amplitude**
- New NR simulations:
  - Circularisation extends at least up to  **$q=3$**  for  **$e \leq 0.2$**

# Construct IMR waveform

Use eccentric PN for inspiral  
(agrees well with NR)

Use a known circular merger (EOB or NR)

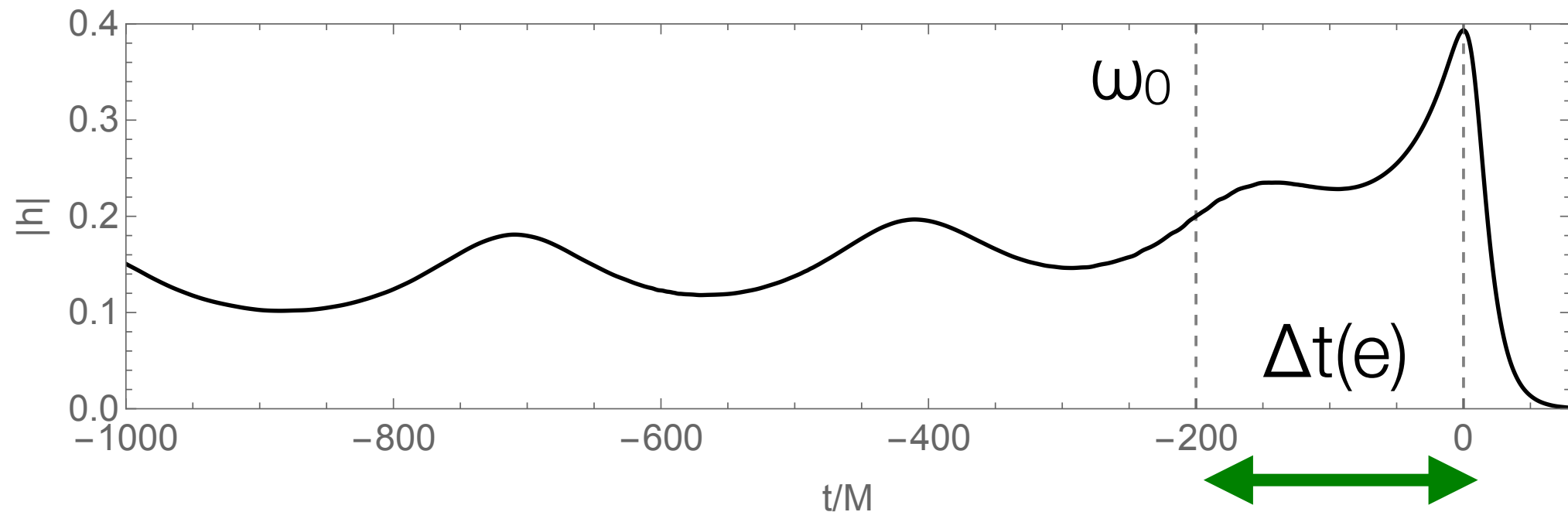


- Make a **best guess**; blend solutions or phenomenological fit
- Blend in **frequency** and **amplitude** of 2,2 mode
- Always validate against NR

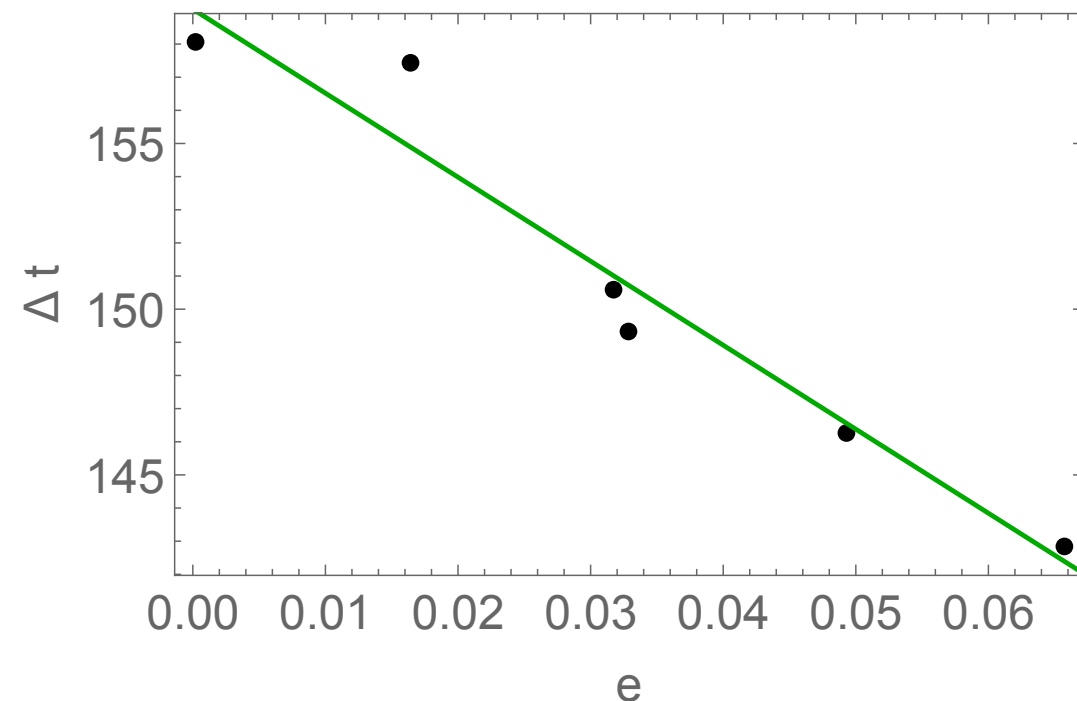


# Where to attach the merger?

- Need **time offset** from  $\omega_0$  to merger peak

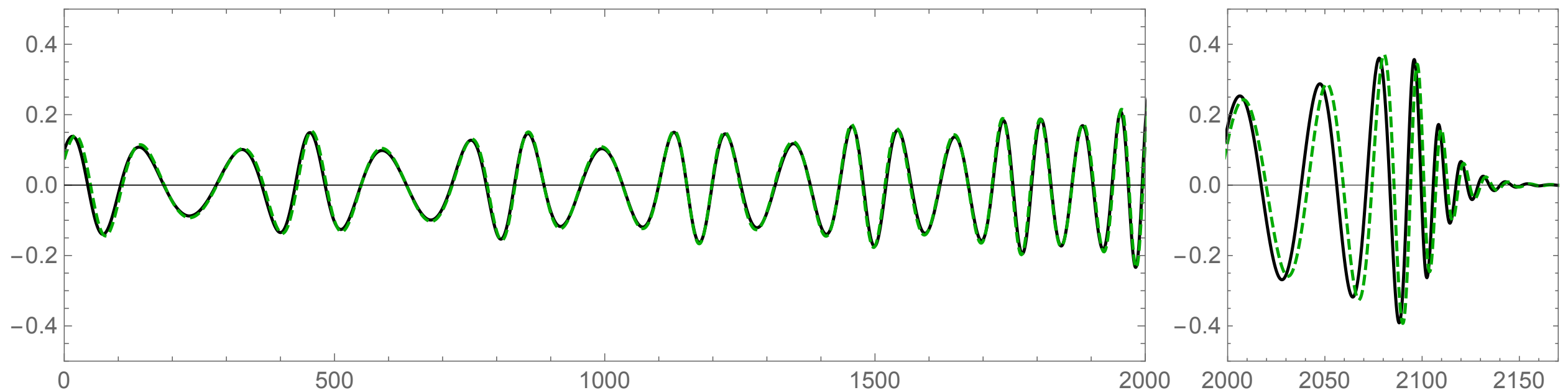


- Calibrate  $\Delta t(e)$  from **NR**
- Can predict **merger time** as function of PN  $e(\omega_0)$  to within  $\pm 2 M$



# Comparison between NR and IMR waveform

- Depending on choice of  $\omega_0$  and **fit window**:
  - Trade-off between dephasing at **merger** and in **early inspiral**
- Example here shows **accurate inspiral** but **dephasing** at merger
- For **short** waveform like GW150914, can instead favour merger

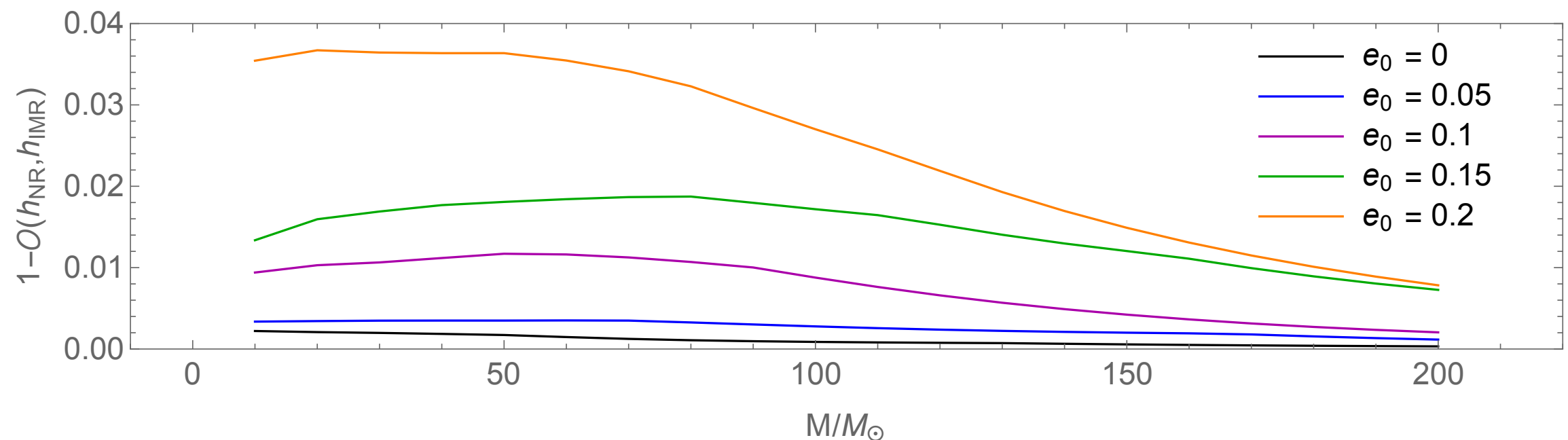


# Unfaithfulness

- Compare NR and PN+NR IMR waveforms in the frequency domain

$$\mathcal{O}(h_1, h_2) = \max_{t_0, \phi_0} \frac{4}{\|h_1\| \|h_2\|} \operatorname{Re} \int_0^\infty \frac{\tilde{h}_1(f) \tilde{h}_2^*(f)}{S_n(f)} e^{i(2\pi t_0 f + \phi_0)} df$$

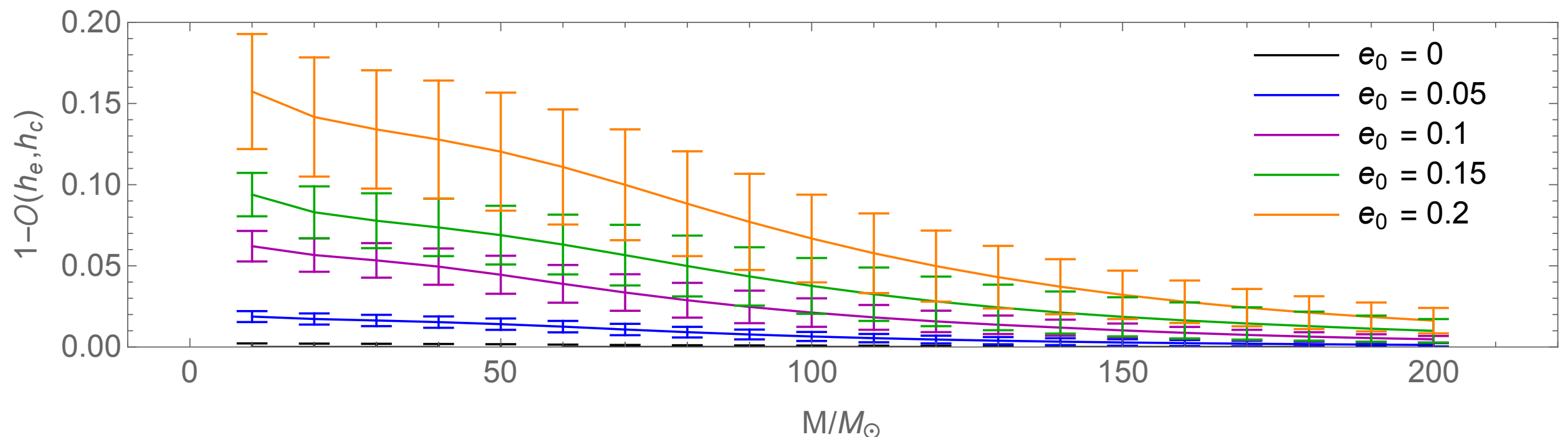
- Rough proxy for how well a GW detector can distinguish waveforms



- NR and IMR agree to **within 4%** unfaithfulness up to  **$e_0 = 0.2$**

# How does eccentricity affect unfaithfulness?

- Unfaithfulness between **NR circular** and **NR eccentric**
- Error bars show error in IMR model



- IMR model should be accurate enough to **distinguish eccentricities**

# Conclusions and outlook

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- **Eccentric inspiral-merger-ringdown BBH waveform model**, non-spinning,  $q \lesssim 0.2$  calibrated to and tested against **Numerical Relativity** simulations
- Agreement with NR:
  - **< 4% unfaithfulness** for  $10 M_{\odot} < M < 200 M_{\odot}$
  - Model errors smaller than differences between eccentric and circular
- Future:
  - Assess implications for **measurement with LIGO**
  - Higher waveform **modes**
  - Improved **transition** from inspiral to merger
  - Add **spin** to PN model