Session Program

Time	Speaker	Talk
14:30	Hinder, Ian	A Review of Recent Developments in Numerical and Analytical Relativity
15:00	Nagar, Alessandro	Gravitational waves from coalescing compact binaries: interfacing analytical and numerical techniques
15:30	Le Tiec, Alexandre	The overlap of numerical relativity, perturbation theory and post-Newtonian theory in the binary black hole problem
		Coffee Break
16:30	Calderón Bustillo, Juan	Accuracy of Complete Hybrid PN/NR Descriptions Of The Gravitational Radiation From Non-Precessing Compact Binaries
17:00	Vano-Vinuales, Alex	Free Hyperboloidal Evolution In Spherical Symmetry
17:30	Puerrer, Michael	Accelerating Parameter Estimation of Gravitational Waves from Black Hole Binaries with Reduced Order Quadratures
18:00	Balmelli, Simone	The description of next-to-leading order spin-spin effects in an Effective-One-Body Hamiltonian

Author	Poster (FISICA Marconi, all week)
Celestino, Juliana	Nonlinear evolution of cylindrical gravitational waves
Ilseven, Ekin	Lattice Boltzmann Model for Numerical Relativity

Lattice Boltzmann Model for Numerical Relativity

E. Ilseven, M. Mendoza

Bona Masso Formalism

First order hyperbolic formulation of Einstein equations

$$\frac{\partial f}{\partial t} + \vec{\beta} \cdot \vec{\nabla} f = S(\alpha, \beta^i, \gamma_{ij}, A_i, B^i_j, D_{kij}, V_i, K_{ij})$$

$$f \in (\alpha, \beta^i, \gamma_{ij}, A_i, B^i_j, D_{kij}, V_i, K_{ij})$$

Lattice Boltzmann Method

A widely applicable CFD method solving any conservation equation

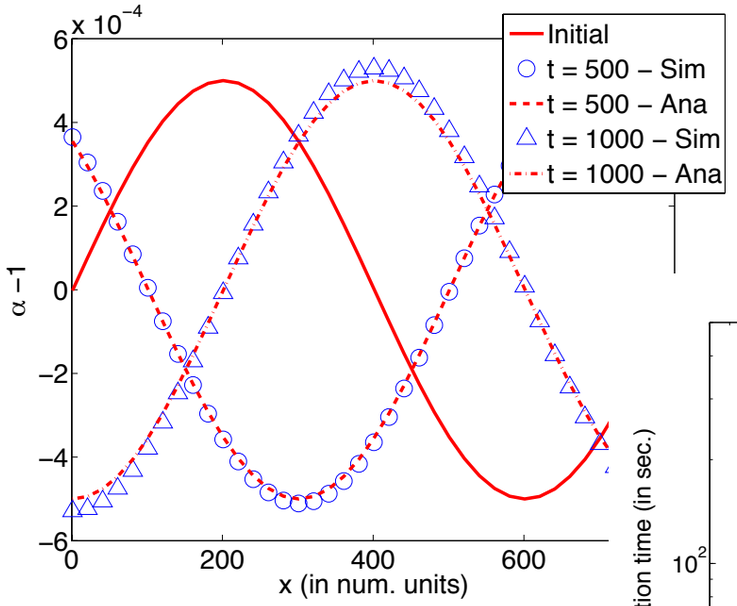
$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f = -\frac{f - f_{\mathrm{eq}}}{\tau} + S$$

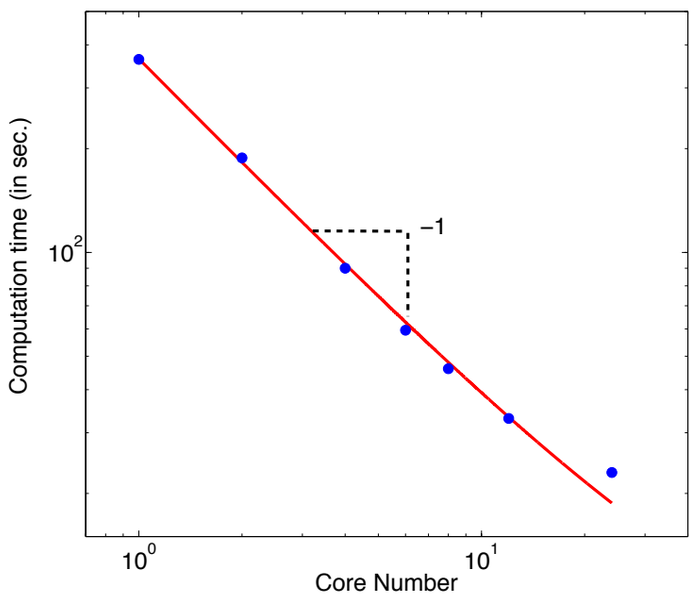
The model is tested through:

- Inflation of flat FLRW universe
- Apples with Apples tests:
  - Gauge Wave
  - Linear Wave

Moreover investigated the advantages of LBM:

- Numerical Diffusivity
- Parallelisation





Nonlinear evolution of cylindrical gravitational waves

- **Juliana Celestino**, Henrique P. de Oliveira, Eduardo L. Rodrigues

- There is currently a vivid expectation of a direct detection of gravitational waves from one or more detectors built specially to this aim. Meanwhile, theoretical studies concerning the wave-forms generated by sources of gravitational radiation and their efficiency are of great interest and constitute a valid effort to understand the role played by the nonlinear character of Einstein equations. The use of numerical techniques is unavoidable if one wants to accomplish any success in exploring at theoretical level the fundamental aspects of gravitational wave emission. In spite of their unphysical character, **cylindrical waves** constitute an interesting nonlinear problem in which both polarization modes are present. We have constructed an **efficient numerical code** to integrate the field equations based on a combination of **Galerkin and pseudospectral methods**. As usual the **code tests** were performed quite satisfactorily, and we have focused on the nonlinear evolution.

A Review of Recent Developments in Numerical and Analytical Relativity

Ian Hinder

Max Planck Institute for Gravitational Physics,
(Albert Einstein Institute), Potsdam, Germany

16th July, 2015
14th Marcel Grossmann Meeting, Rome

Overview

- The Numerical Relativity field
- Analytic inspiral-merger-ringdown waveform families
- Edges of NR parameter space
- A selection of recent results

The Numerical Relativity field

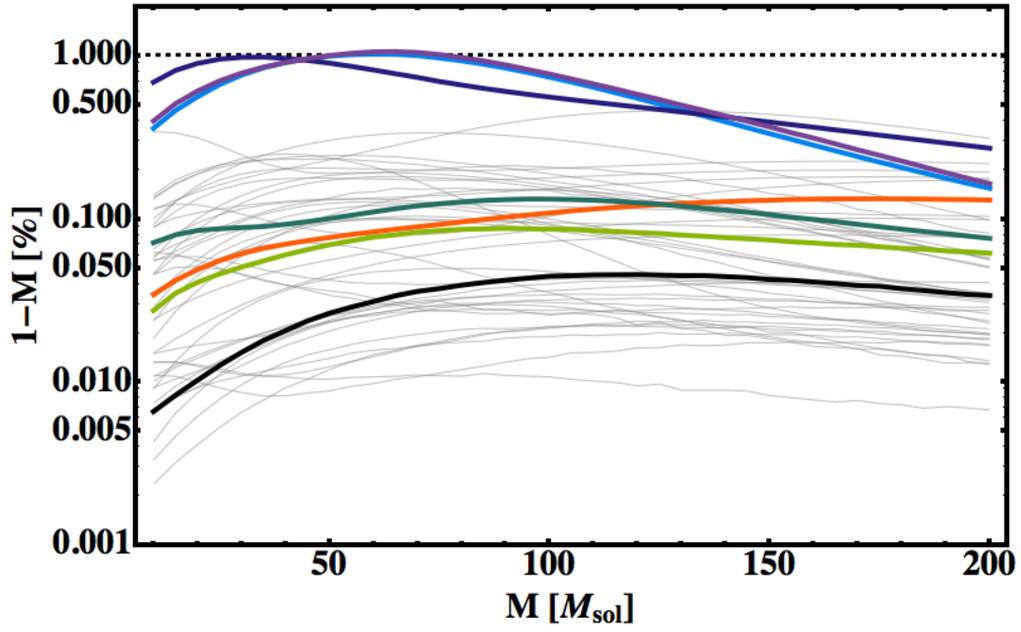
- **118 papers** related to Numerical Relativity in the past year
- NR **waveform catalogues**:
  - NINJA 1, 23 waveforms, 2009
  - NINJA 2, 60 hybrid waveforms, 2012
  - NRAR, 25 waveforms, 2013
  - SXS: Originally 174 waveforms. Now 201. 2013 onwards.

Latest "Analytic" waveform models

- Effective-One-Body variants:
  - Precessing **SEOBNRv3** (Taracchini et al. 2014) and **Damour and Nagar, 2014** (recalibrated June 2015)
  - System of ODEs solved via **numerical integration**
    - Computational cost can be decreased via use of **reduced-order modelling** (see Michael Pürrer's talk today)
- Precessing **PhenomP** - Hannam et al. 2014:
  - "Simple" **closed-form** frequency-domain expression for h; single-spin and stationary-phase approximations

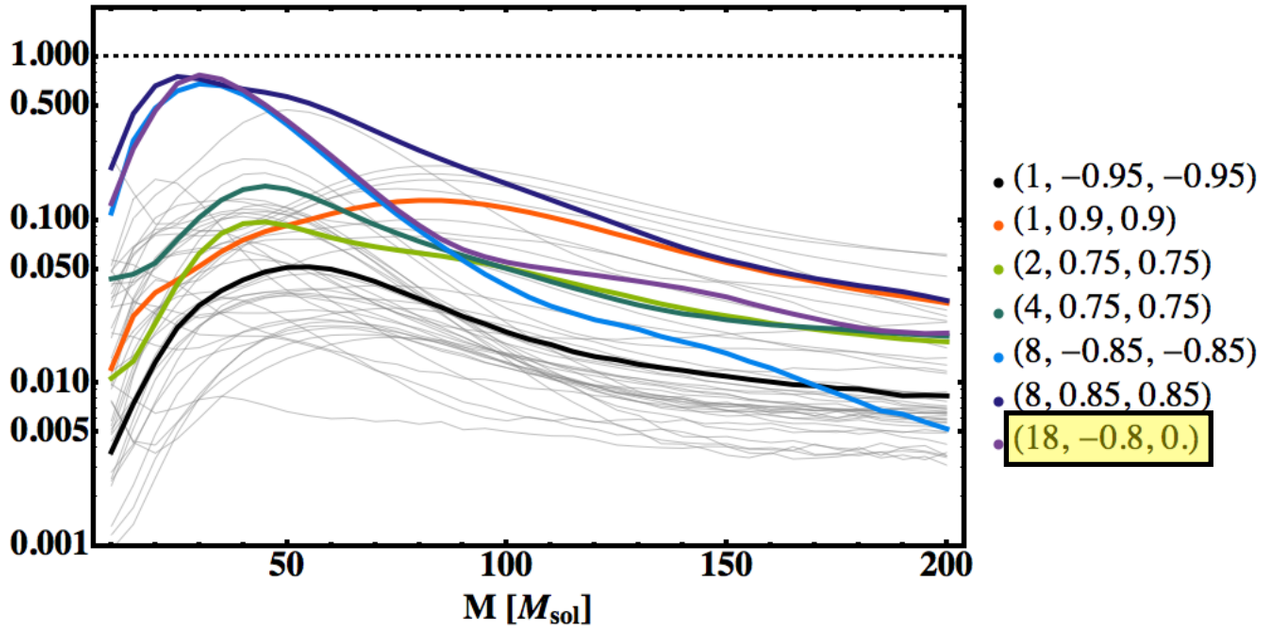
Latest "Analytic" waveform models

- **PhenomD** (in preparation, **PRELIMINARY**):
  - 19 calibration points in q, S space including **q=18, χ_1=-0.8**
  - **Mismatch < 1%** over calibration range for 47 waveforms



-aLIGO zdethp Noise curve

-Lower frequency 10Hz



-Early aLIGO Noise curve

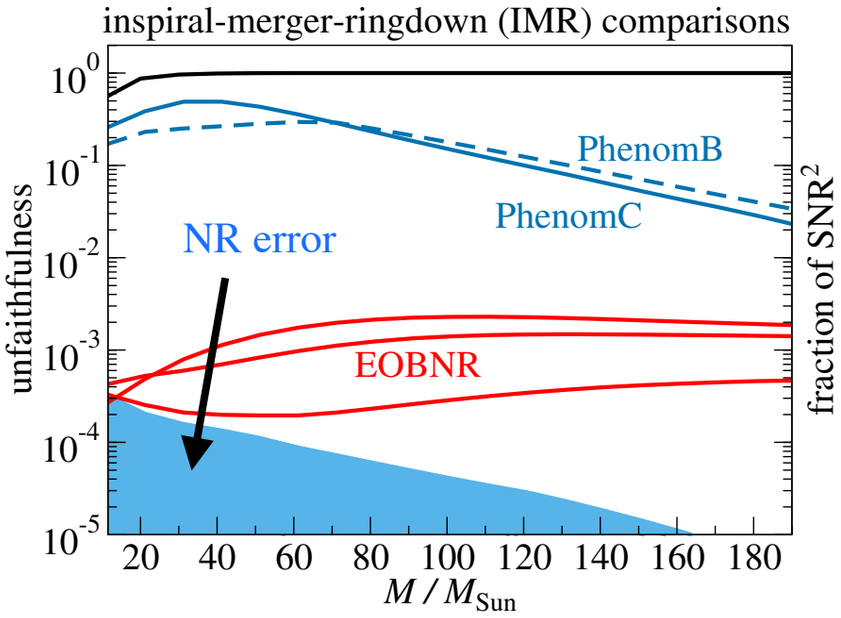
-Lower frequency 30Hz

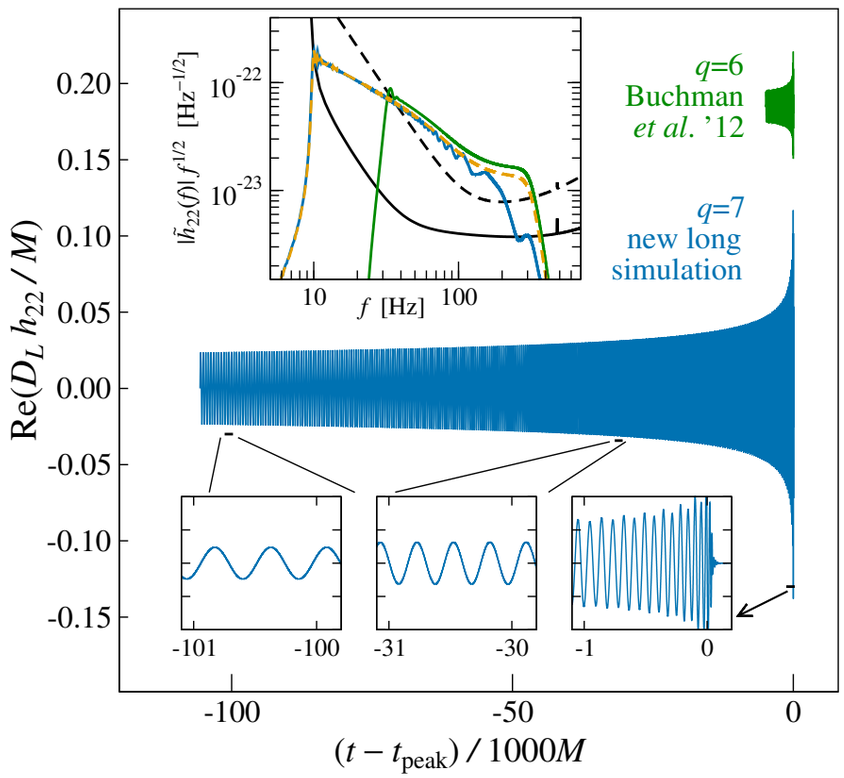
Edges of NR parameter space

- Number of **cycles** / lowest frequency
- High **spin**
- High **mass ratio**

Longest NR waveform

- Szilagyi et al., 2015 (Numerical relativity reaching into post-Newtonian territory: a compact-object binary simulation spanning **350 gravitational-wave cycles**)
- **SpEC** code
- Mass ratio **q=7**
- Separation D = 27 M
- NR waveform covers aLIGO band for $M_1+M_2 >$ **45.5 M⊙**





- Previous longest: **60 cycles**
- Pan et al., 2013: with NR errors at the time, 30 and 50 cycle EOB waveforms were **indistinguishable to aLIGO**

Lowest orbital frequency

- Lousto and Zlochower, 2013 (Exploring the Outer Limits of Numerical Relativity)
- **LazEv** code
- Equal mass, non-spinning
- Started **4000 GW cycles** before merger
- Evolved for 3 cycles (no merger) at **D = 100 M**

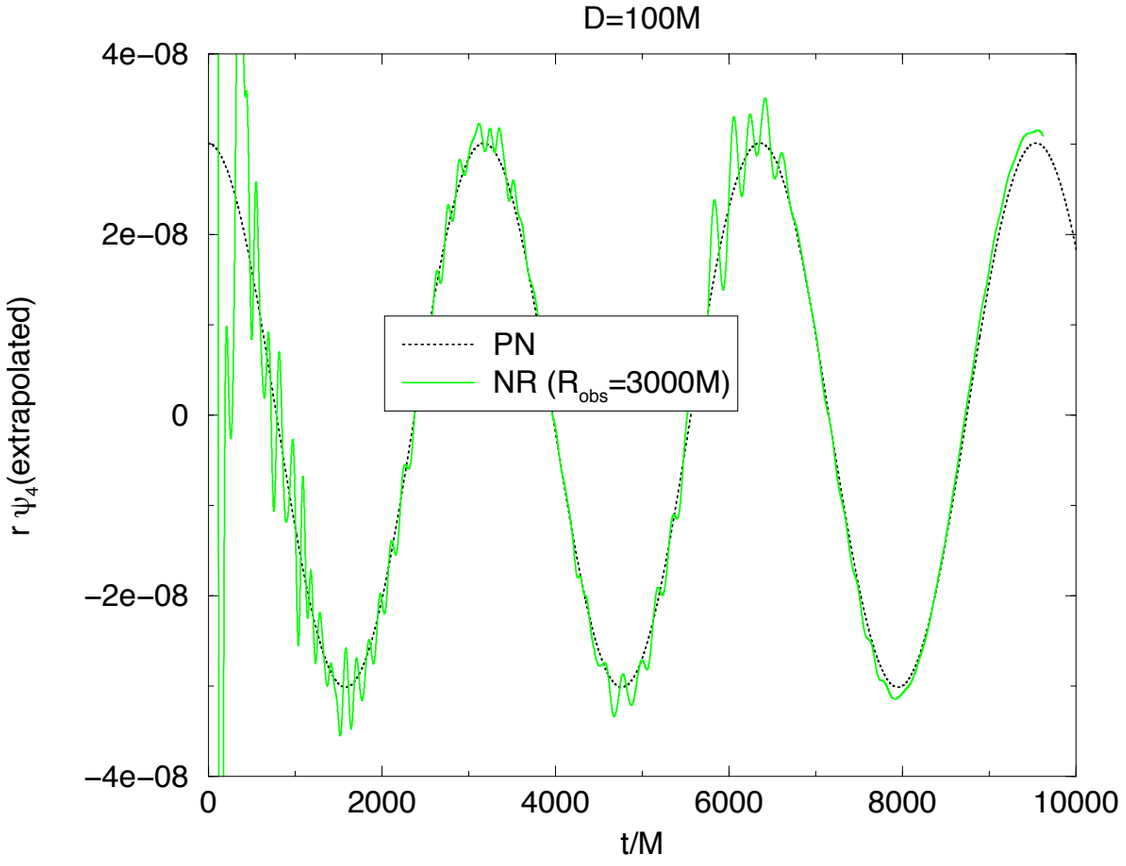
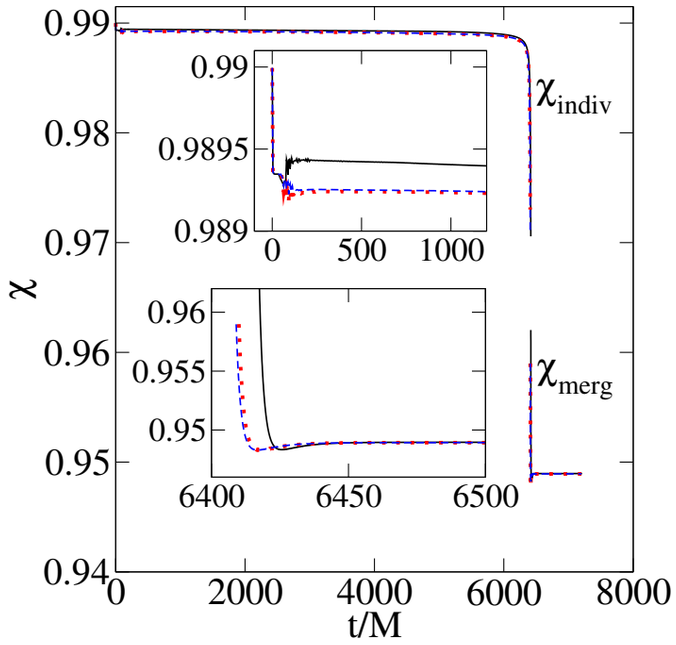
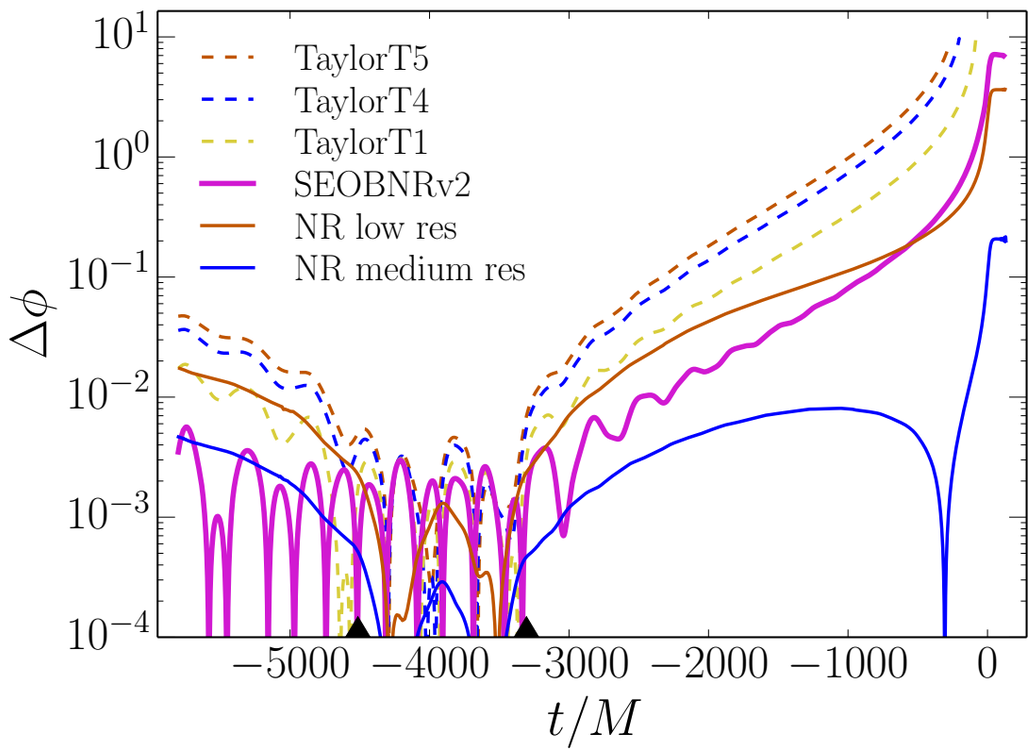


TABLE IV: Energy and angular momentum radiated per orbit (initial and second).

D	$\delta m/M_{\rm num}$	$\delta m/M_{\rm PN}$
20M	$(5.68 \pm 0.02) \times 10^{-5}$	$5.43 - 5.62 \times 10^{-5}$
50M	$(2.4 \pm 0.1) \times 10^{-6}$	$2.52 - 2.53 \times 10^{-6}$
100M	$(2.3 \pm 0.4) \times 10^{-7}$	$2.36 - 2.36 \times 10^{-7}$
D	$\delta J/M^2_{\rm num}$	$\delta J/M^2_{\rm PN}$
20M	$(5.39 \pm 0.01) \times 10^{-3}$	$5.20 - 5.30 \times 10^{-3}$
50M	$(8.9 \pm 0.1) \times 10^{-4}$	$9.16 - 9.18 \times 10^{-4}$
100M	$(2.4 \pm 0.4) \times 10^{-4}$	$2.39 - 2.39 \times 10^{-4}$

Highest spins



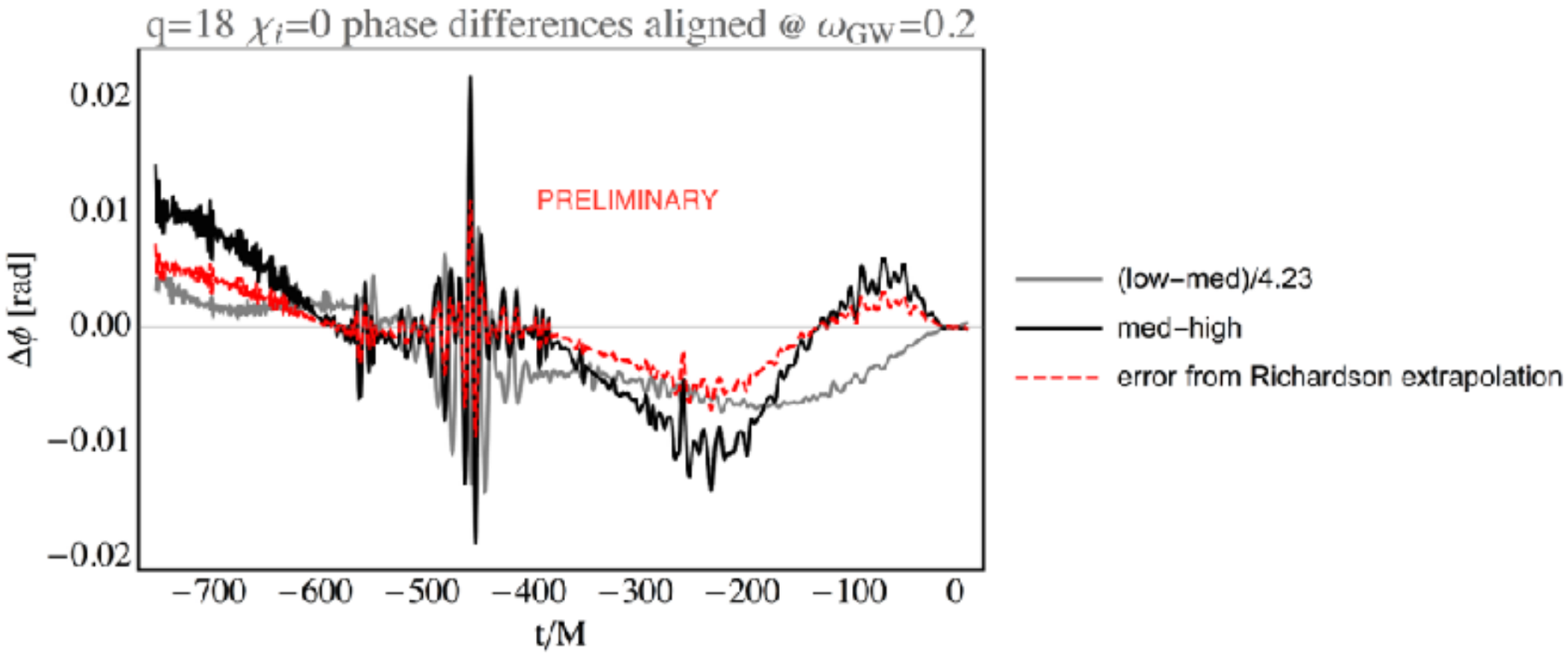


- Scheel at al. 2014 (Improved methods for simulating nearly extremal binary black holes)
- **Superposed Kerr-Schild initial data** to exceed the Bowen-York limit
- **SpEC** code with improved **numerical methods** and **code optimisation**

- Simulations:
  - q=1, aligned χ_1=χ_2=**0.994**, 50 cycles (**highest spin** ever simulated)
  - q=1, aligned χ_1=χ_2=**0.99**, 50 cycles
  - q=1.5, χ_1=**0.99** (aligned), χ_2=0.2 misaligned, 46 cycles
- NR phase error smaller than difference with PN and EOB approximants for χ=0.994

Highest mass ratio

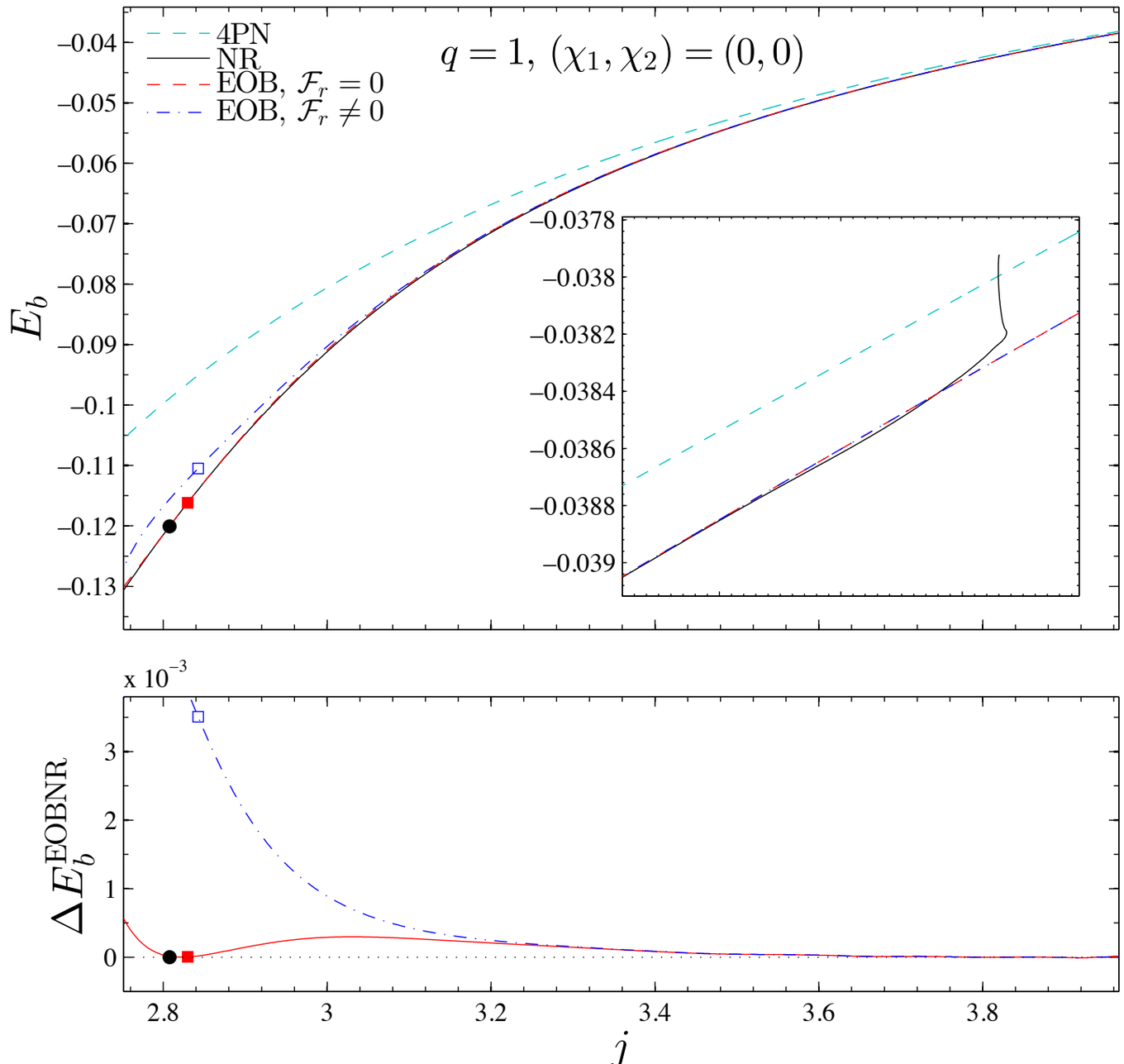
- **q = 100** (1 orbit), Nakano et al., 2011 (Intermediate-mass-ratio black hole binaries II: Modeling Trajectories and Gravitational Waveforms)
- **q = 18**, 7 orbits, non-spinning (Varma et al., 2014)
- [Preliminary] **q = 18**, 8 orbits, **χ_1 = -0.8**



Recent Results

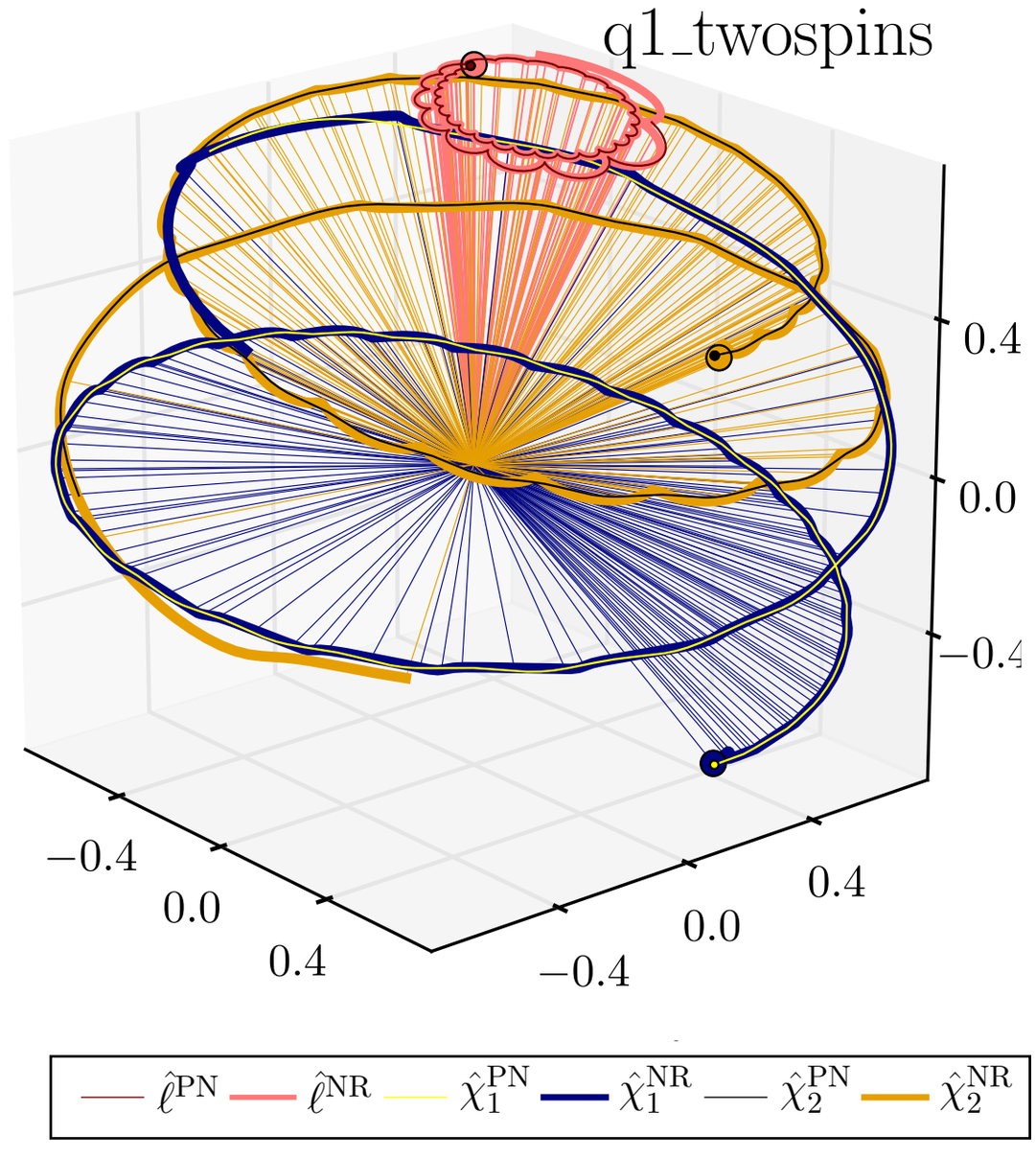
Energy and Angular Momentum Comparison

- Nagar et al., 2015 (Energetics and phasing of nonprecessing spinning coalescing black hole binaries)
- Radiated **E and J** computed from NR simulations using **SpEC** and **Llama** codes and compared with EOB
- See talk by **Alessandro Nagar** (next)



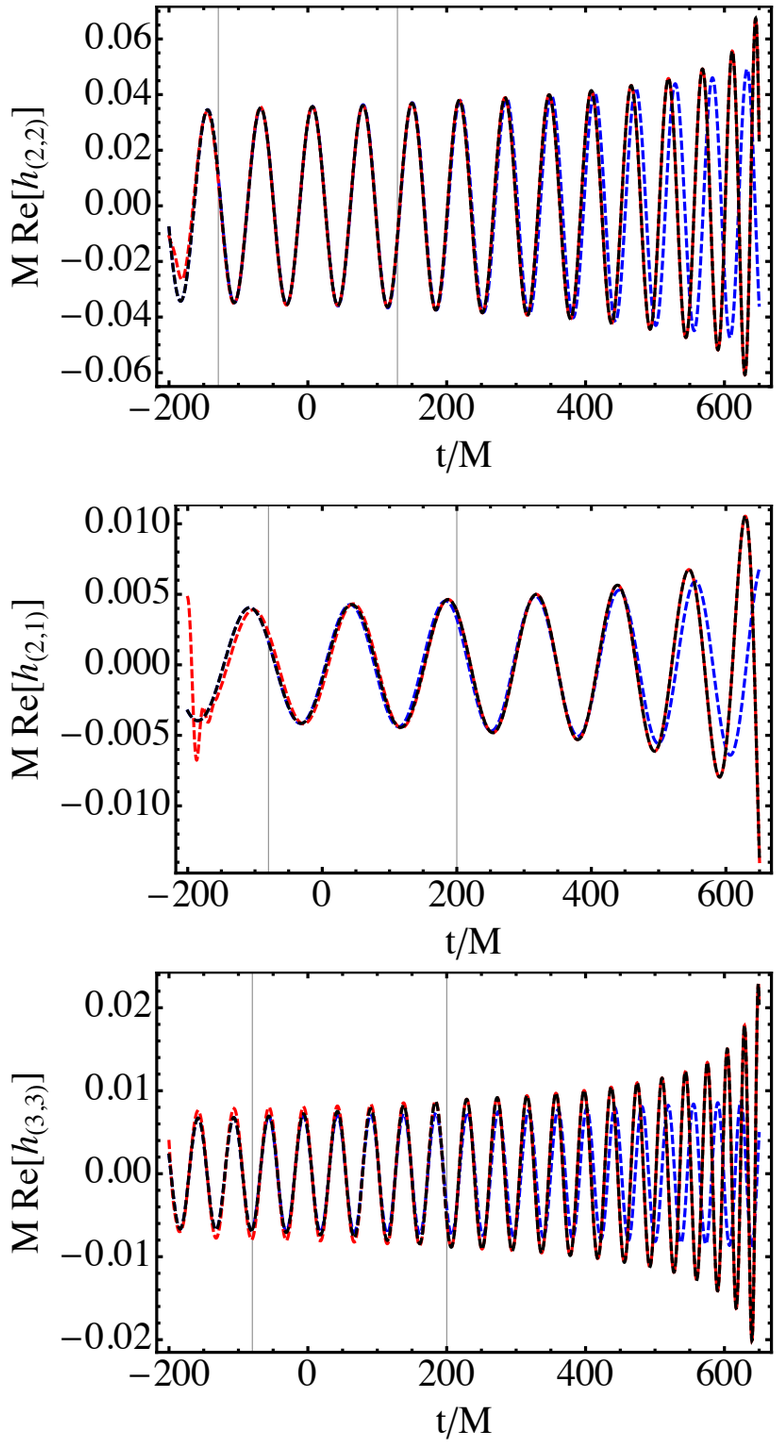
Precession Dynamics

- Ossokine et al., Jan 2015
- 37 **precessing** simulations (from SXS catalogue)
- Mass ratios up to 8, up to **2 precession cycles** (>~ 60 wave cycles)
- **Orbital angular momentum** and **spins** agree between NR and PN to within **1 degree** during inspiral and within **5 degrees** at merger



Sub-dominant modes

- Calderón Bustillo et al. 2015 (Comparison of subdominant gravitational wave harmonics between post-Newtonian and numerical relativity calculations and construction of **multi-mode hybrids**)
- Provides an algorithm for constructing hybrids with **higher order modes**
- Uses waveforms from **SXS** and **BAM** codes, **q=8** and **q=18**
- See Juan Calderón Bustillo's **talk after coffee break** for an update



2,2

2,1

3,3

Spin "flip-flop"

- Lousto and Healy, 2015 (Flip-Flopping Binary Black Holes)
- Lousto and Healy, 2015 (Spin flips in generic black hole binaries)
- **LazEv** code
- Reproduction of **PN spin dynamics** using NR

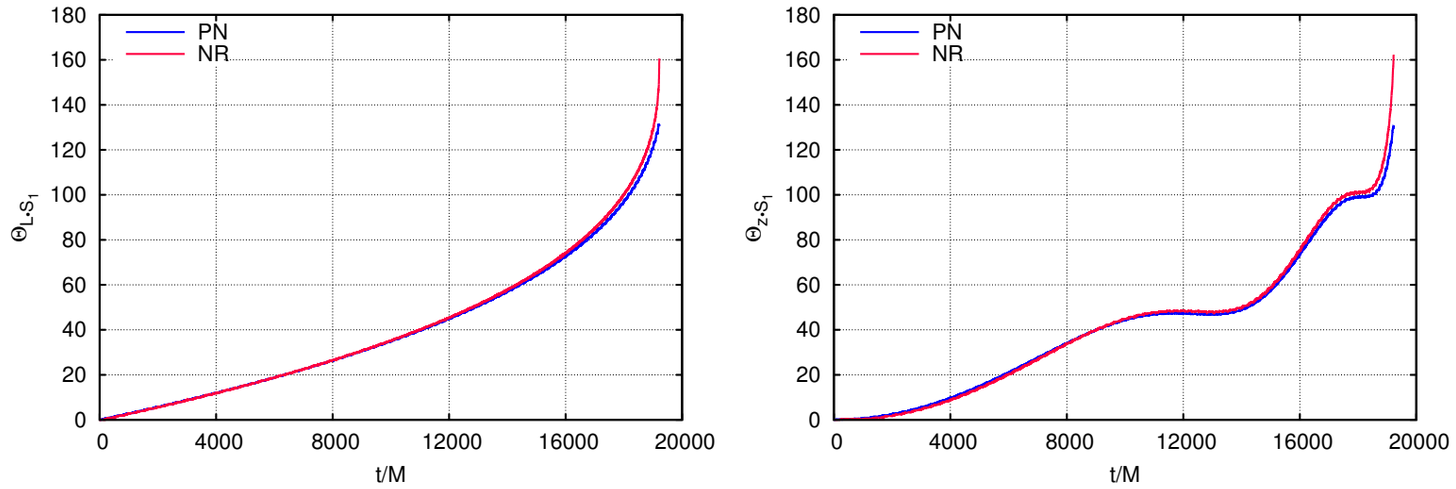
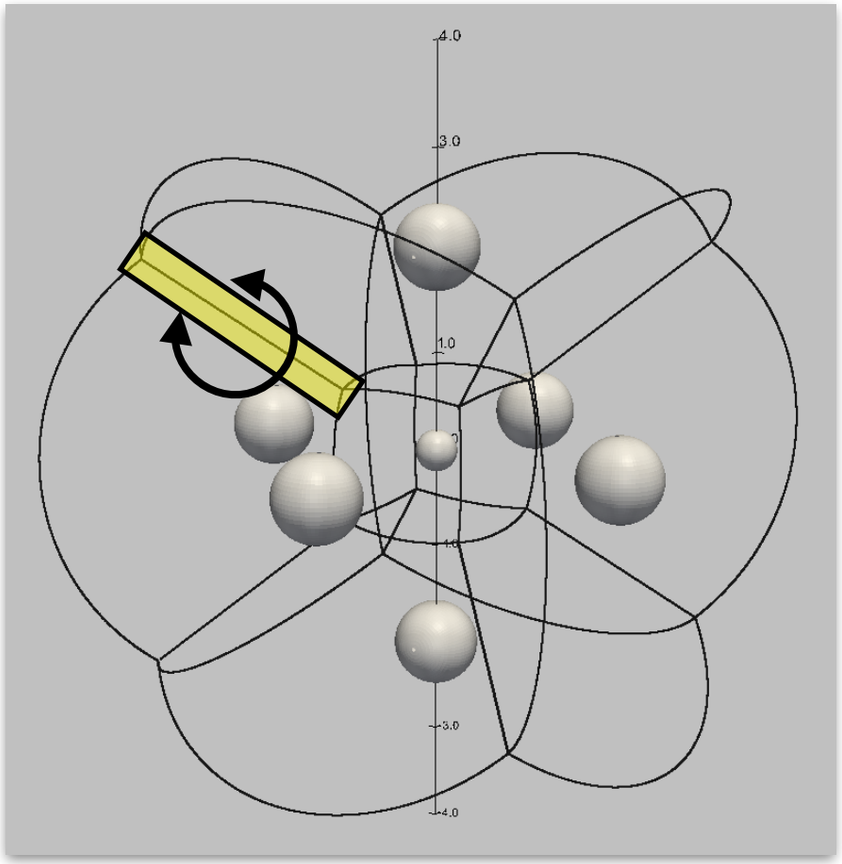


FIG. 2. The angle between the spin of the secondary black hole $\vec{S}_1$ with respect to the orbital angular momentum $\vec{L}$ (left) and with respect to the fixed z-axis (right). For comparison we also plot the 3.5PN prediction.

Cosmology: Black Hole Lattices

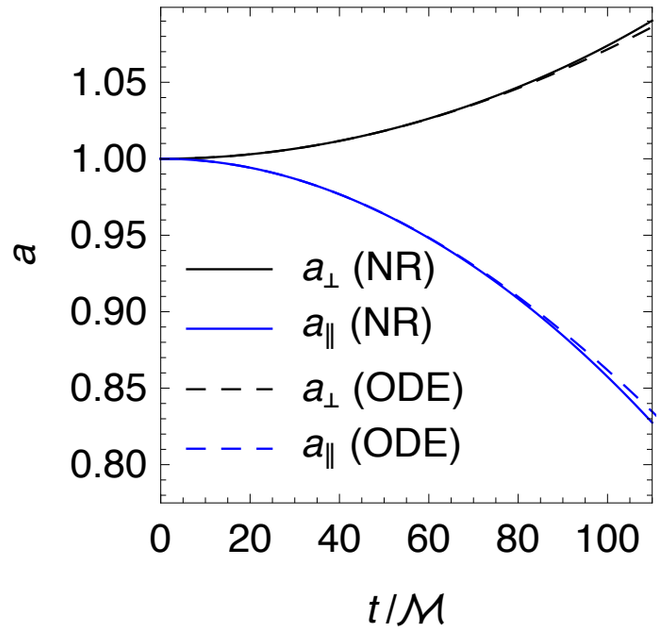
- Korzyński, IH, Bentivegna 2015 (On the vacuum Einstein equations along curves with a **discrete local rotational and reflection symmetry**)



Lattice cell edges after stereographic projection to R^3

- Claim in literature: Einstein equations **reduce to ODEs** for metric and electric Weyl on edge:

$$\frac{\ddot{a}_\parallel}{a_\parallel} = \frac{2}{3}E_+,$$
$$\frac{\ddot{a}_\perp}{a_\perp} = -\frac{1}{3}E_+.$$
$$\dot{E}_+ = -3\frac{\dot{a}_\perp}{a_\perp}E_+$$



but full 3D **numerical relativity evolution disagrees** (NR error bars negligible in plot)

Cosmology: Black Hole Lattices

- Careful re-examination of ODE reduction argument reveals **incorrect symmetry assumption**. Derived missing term in ODE:

$$\frac{\ddot{a}_\parallel}{a_\parallel} = \frac{2}{3}E_+, \qquad \frac{\ddot{a}_\perp}{a_\perp} = -\frac{1}{3}E_+. \qquad \dot{E}_+ = -3\frac{\dot{a}_\perp}{a_\perp}E_+ \boxed{-\tfrac{3}{2}U_{11},} \qquad U_{11} = (K_{ij;k}{}^{k} - K_{ik;j}{}^{k})\,\mathbf{e}_1^i\,\mathbf{e}_1^j$$

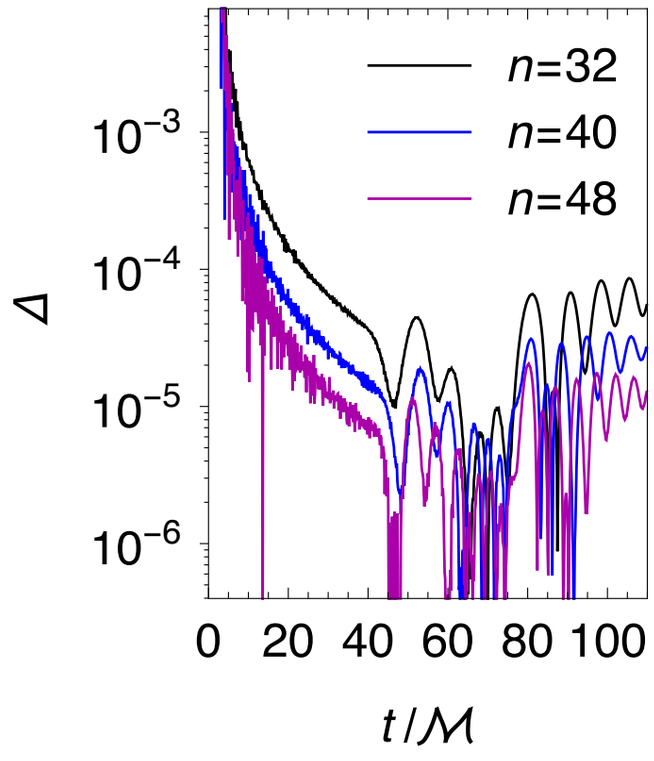
- NR shows that this **perfectly accounts** for the anomaly

$$-\frac{2}{3}\mathcal{A} = 1.00000(3)\,U_{11}$$

- A tedious Mathematica calculation eventually proved analytically that U_{11} has a non-zero **3rd time derivative** at t=0. Discrepancy grows as t^6.

- Verified value with NR to 4 digits of precision

$$\left.\frac{\partial^3 U_{11}}{\partial t^3}\right|_{t=0,\phi=0} = \begin{cases} 4.3015(4)\times 10^{-12}\mathcal{M}^{-6} & \text{Numerical} \\ 4.30113\times 10^{-12}\mathcal{M}^{-6} & \text{Analytic}\,. \end{cases}$$



(c) $\Delta \equiv 1 - (-2/3\mathcal{A})/U_{11}$ computed from NR at several resolutions.

Summary

- NR BBH simulations up to **q=18**, or **350 cycles**, or **S/m^2=0.994**. Pick any **one**!
- Recent results on:
  - Energy and Angular Momentum Comparison
  - Precession Dynamics
  - Sub-dominant modes
  - Spin "flip-flop"
  - Cosmology: Black hole lattices